New Weights for Estimating Normal Surface in the Triangular Mesh

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Abstract: The normal vector at a node on the discrete surface is not unique, when a surface discretized by triangular elements. A widely used technique to estimate the normal vector at a giving node is a weighted average of the exact normal of the surrounding triangular elements. In this paper, two weight factors proposed to estimate the normal vector at a node in the triangular mesh. The numerical comparisons are performed between the new weights and with some existing weights for the two knowing geometrical surface.

Keywords: Averaging Normal Vector, Triangular Mesh, Weight Factors, Toroidal Surface

1. Introduction

One of the geometric properties of discrete surface is calculating accurate normal vector at a point on a surface as it is important in many computer graphics, curvature estimation, tangent plane computations, and geometric modelling applications. The surface triangulation is widely used to discretize a surface in computer applications for example smooth shading (Gouraud, 1971 and Phong, 1975) and in numerical simulation methods such boundary and finite element methods. More applications of the can be found in direct rendering of points in computer graphics (Schaufler and Jensen, 2000). Therefore, it is necessary to have information about the normal vectors at the nodes.

In general, each node in triangular mesh connected to more than one triangle. In other word, a number of triangles are shared in a node. The triangular element has a unique normal vector at all points except at the vertices. Therefore, some estimation needs to find accurate normal surface at the vertices of the triangles. Many researchers have attempted to estimate the normal vectors of discrete points by fitting smooth parametric surfaces (Hoffman and Jain 1987; Yang and Lee 1999; Zhang et al., 2001) or by generating triangular surface models (Milroy et al., 1997; Huang and Menq, 2001; Woo et al., 2002). In any case, the estimation procedure would involve at least the following two main steps. Identify the applicable neighboring points for estimating the normal vector and estimate the normal vector based on points in the local neighborhood. These two steps are main feature of the methods in this research (sections 2 and 3)

2. Problem Statement

In this section, we illustrate some weights for estimating the normal vector of a node in the triangular mesh by using the exact normal vectors of the surrounded triangular elements. Suppose we have a surface S discretized into M triangles with N vertices (nodes). Let $\boldsymbol{p}$ be an arbitrary node on the surface surrounded by the nodes $\boldsymbol{r}_1$, $\boldsymbol{r}_2$, …, $\boldsymbol{r}_k$ and by the triangular elements $f_1, f_2, ..., f_k$ with their exact normal vectors are $\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2, ..., \hat{\boldsymbol{n}}_k$ respectively; and let $\boldsymbol{n_p}$ be the normal surface at the node $\boldsymbol{p}$ as illustrated in the figure 1. The main goal is to find the weight factors w_i, i=1, 2, …, k such that

$$\boldsymbol{n}_p = \frac{\sum_{i=1}^{k} w_i \hat{\boldsymbol{n}}_i}{\left\| \sum_{i=1}^{k} w_i \hat{\boldsymbol{n}}_i \right\|} + \boldsymbol{e}, \tag{1}$$

where $\boldsymbol{e}$ represents the error made and it must tend to zero by approaching the size of the triangular elements to zero. Many studies have proposed a formula of calculating w_i to estimate the normal vectors by using polygon surface model. For example, Meek and Walton, (2000); and Taubin (1995) used the area of the triangle,

$$w_i = \frac{1}{2} \left\| (\boldsymbol{r}_i - \boldsymbol{p}) \times (\boldsymbol{r}_{i+1} - \boldsymbol{p}) \right\|. \tag{2}$$

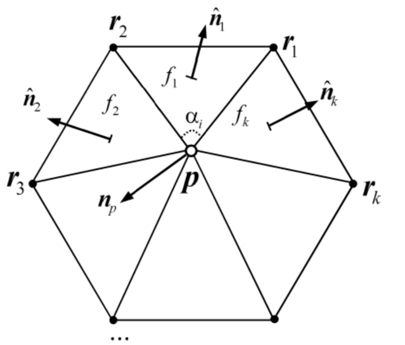

Figure (1): Sketch of the surrounding triangular elements to the arbitrary node $\boldsymbol{p}$.

Equal weights (equation 3) suggested by Gouraud (1971), and centroid of the triangle (equation 4) suggested by Chen and Wu (2004),

$$w_i = 1, \tag{3}$$

$$w_i = \frac{1}{\left\| \boldsymbol{g}_k - \boldsymbol{r}_i \right\|^2}, \quad \text{where} \qquad \boldsymbol{g}_j = \frac{\sum_{\boldsymbol{r}_j \in f_k} \boldsymbol{r}_j}{3}. \tag{4}$$

The inverse of equation (1), the angle α_i in the triangle $\Delta \boldsymbol{r}_i\, \boldsymbol{p}\, \boldsymbol{r}_{i+1}$ (see figure 1) and the circumference of the triangle are all used by Ubach, et al. (2013).

3. New Weight Factors

In this section, we suggest two new weight factors by transforming the coordinates of arbitrary node $\boldsymbol{p}$ and it's surrounded nodes to the local coordinate system XYZ with origin O at the node $\boldsymbol{p}$ and its Z-axis along the normal direction $\boldsymbol{N}_p^0$ at the node $\boldsymbol{p}$. where $\boldsymbol{N}_p^0$ is calculated by one of the methods in section 2. Suppose that w_i=β_i is the angle between the plane Z=0 and the triangle f_i as can be seen in figure 2a. Second weigh factor is w_i=v_i where v_i is the volume of the pyramid that generated by projecting the triangle f_i on the plane Z=0. Therefore, if we denote the vertices of the triangle fi in the local coordinates by $\boldsymbol{r}'_i, \boldsymbol{r}'_{i+1}, O$, then the base vertices of the pyramid are $\boldsymbol{r}'_i, \boldsymbol{r}'_{i+1}, \boldsymbol{r}''_i, \boldsymbol{r}''_{i+1}$, where $\boldsymbol{r}''_i, \boldsymbol{r}''_{i+1}$ are projected points of $\boldsymbol{r}'_i$, $\boldsymbol{r}'_{i+1}$, on the plane Z=0, respectively (see figure 2b). Therefore, we suggest the new normal vector at the node origin in local coordinate as,

$$\boldsymbol{N}_{\boldsymbol{p}}^{j+1} = \frac{\sum_{i=1}^{k} w_i \boldsymbol{N}_i}{\left\| \sum_{i=1}^{k} w_i \boldsymbol{N}_i \right\|}, j = 1, 2, \quad (5)$$

Where $\boldsymbol{N}_i$ are the actual normal vectors for the face f_i, i=1, 2, …, k. we stop the iteration in equation(5), when $\left\| \boldsymbol{N}_{\boldsymbol{p}}^{j+1} - \boldsymbol{N}_{\boldsymbol{p}}^{j} \right\| < \varepsilon, j = 0,1,...$ and we choose $\varepsilon=10^{-5}$ in this research. Thus, $\boldsymbol{N}_{\boldsymbol{p}}^{j+1}$ is estimated normal vector in the local coordinate. It should be transom it to the global coordinates.

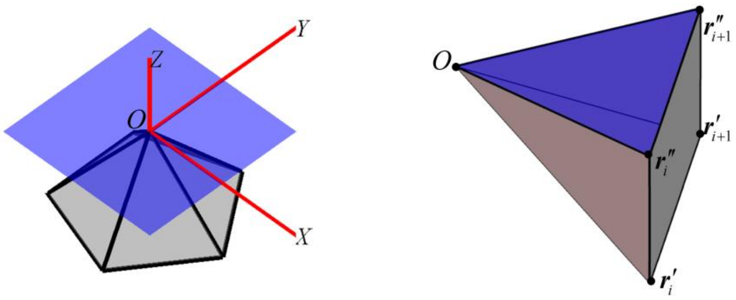

Figure 2: (a) Sketch of the surrounding triangular elements to the node $\boldsymbol{p}$ at the local coordinates system O-XYZ. (b) Pyramid with origin O is apex and base vertices are $\boldsymbol{r}'_i$, $\boldsymbol{r}'_{i+1}$, $\boldsymbol{r}''_i$, $\boldsymbol{r}''_{i+1}$.

4. Numerical Examples

Two typical surfaces are meshed by planer triangular elements. Initially 500 nodes on the surfaces considered. The root-mean-square error ($RMSE$) angle in degrees is calculated by the equation (6) for each weights presented in the section 2 and 3;

$$RMSE = \sqrt{\sum_{j=}^{n} \frac{d_j^2}{n}}, \quad (6)$$

where

$$d_j = \arccos(n_j^{appro}.n_j^e).\frac{180}{\pi}, \quad (7)$$

where n_j^{appro} and n_j^e are estimated and exact norm vectors at the node j, respectively. All approximated methods must converge to the exact normal vector by increasing the number of the nodes on the surfaces.

The first surface is a mode of the spherical harmonic, which represented in spherical coordinates (ρ, θ, φ) as,

$$(\rho - 4)^2 = \cos 2\varphi \sin\theta, \quad (8)$$

where $0 \le \varphi \le 2\pi$, $0 \le \theta \le \pi$ (see figure 3a).

(a) (b)

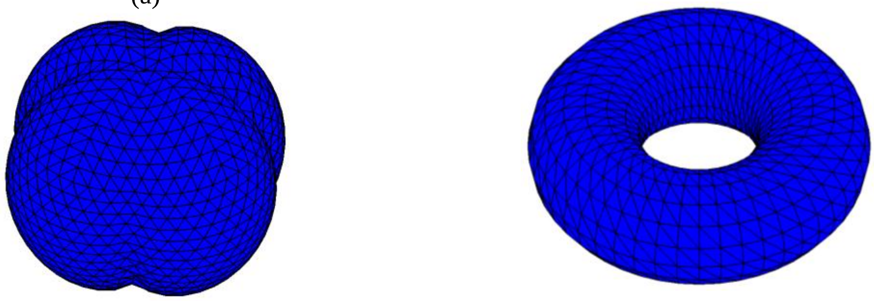

Figure 3: (a) Spherical harmonic surface showed in equation (8) (b) Toroidal surface expressed in equation (9).

The weights in section 2 and 3 are used to calculate the normal vector and they compared with the exact normal vector. In Figure 4, it is clear The *RMSE* for all weighs in section 2 and 3 are reduce with increasing node numbers, but the reduction in the new weights in section 3 are faster.

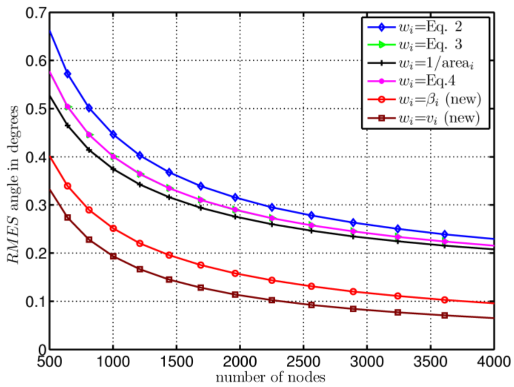

Figure 4: The *RMSE* values of normal estimation for the surface in equation (8).

The second surface is the torus, which given by

$$\begin{aligned} x(\theta,\varphi) &= (r_1 + r_2\cos\theta)\cos\varphi \\ y(\theta,\varphi) &= (r_1 + r_2\cos\theta)\sin\varphi \qquad 0 \le \theta,\varphi \le 2\pi, \\ z(\theta,\varphi) &= r_2\sin\theta \end{aligned} \tag{9}$$

where r_1=0.8 and r_2=0.4 as showed in figure 3b. The points in on the torus can be considered as a parabolic, elliptical or hyperbolic point. Therefore, this type of the surface is good for investigation.

Figure 5 shows the *RMSE* errors for the points on the torus surface, in the similar for the pervious surface all the estimated normal vectors in all methods in section 2 and 3 are converge to the exact normal by increasing number of nodes, but, the methods in section 3 are converge faster.

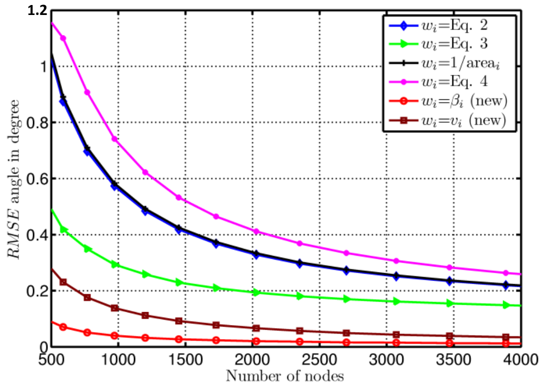

Figure 5: The *RMSE* values of normal estimation for the toroidal surface in (9)

5. Conclusions

The direct method for estimating the normal vector at a point on the surface in the triangular mesh is the weighted average of the normal vectors of the surrounding triangles. Two new weights are proposed for estimating the normal vector at a point on the discrete surface. According to the numerical examples, we conclude that the new weights converge faster than the previous existing weights.

References

Chen, S. G. & Wu, J. Y. (2004). Estimating normal vectors and curvatures by centroid weights. *Computer Aided Geometric Design*, 21(5), 447-458.

Gouraud, H. (1971). Continuous shading of curved surfaces. *IEEE Transactions on Computers*, 100(6), 623-629.

Hoffman, R., & Jain, A. K. (1987). Segmentation and classification of range images. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, (5), 608-620.

Huang, J., & Menq, C.H. (2001). Automatic data segmentation for geometric feature extraction from unorganized 3-D coordinate points. *IEEE Transactions on Robotics and Automation*, 17(3), 268-279.

Meek, D. S., & Walton, D. J. (2000). On surface normal and Gaussian curvature approximations given data sampled from a smooth surface. *Computer Aided Geometric Design*, 17(6), 521-543.

Milroy, M. J., Bradley, C., & Vickers, G. W. (1997). Segmentation of a wrap-around model using an active contour. *Computer-Aided Design*, 29(4), 299-320.

Phong, B. T. (1975). Illumination for computer generated pictures. *Communications of the ACM*, 18(6), 311-317.

Schaufler, G., & Jensen, H.W. (2000). Ray tracing point sampled geometry. *Rendering Techniques 2000: 11th Eurographics Workshop on Rendering* (319-328).

Taubin, G. (1995). Estimating the tensor of curvature of a surface from a polyhedral approximation. *In Computer Vision*, 1995. *Proceedings., Fifth International Conference on* (902-907). IEEE.

Ubach, P.A., Estruch, C., & Garcia-Espinosa, J. (2013). On the interpolation of normal vectors for triangle meshes. *International Journal for Numerical Methods in Engineering*, 96(4), p.247-268.

Woo, H., Kang, E., Wang, S., & Lee, K.H. 2002. A new segmentation method for point cloud data. *International Journal of Machine Tools and Manufacture*, 42(2), 167-178.

Yang, M., & Lee, E., (1999). Segmentation of measured point data using a parametric quadric surface approximation. *Computer-Aided Design*, 31(7), 449-457.

Zhang, Y.L., Yeo, K.S., Khoo, B.C., & Wang, C. (2001). 3D jet impact and toroidal bubbles. *Journal of Computational Physics*, 166(2), 336-360.