On Soft Semi-Open Sets and Soft Semi-Continuity in Fuzzifying Soft Topological Spaces

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Abstract: In this paper, a new class of soft open sets in fuzzifying soft topological spaces called soft semi open sets is introduced and its fuzzifying soft topological properties is studied and investigated. Moreover, we aim to introduce and study the concepts of soft semi neighborhood system, soft semi interior, soft semi closure and soft semi boundary operators, in fuzzifying soft topological spaces. Finally, the concept of soft semi-continuity is defined and studied.

Keywords: Fuzzifying Soft Topology, Fuzzifying Soft Semi-Open Sets, Fuzzifying Soft Semi-Continuity

1. Introduction

The connotation of a fuzzy set was initiated by Zadeh (1965) in his classical paper of 1965. Three years later, Chang (1968) gave the definition of fuzzy topology. Ying (1991) used the semantic method of continuous valued logic to propose the so-called fuzzifying topology as a preliminary of the research on bifuzzy topology and elementary developed topology in the theory of fuzzy sets from completely different direction. The concept of soft set theory has been introduced by Molodtsov (1999) this set designed to solve the sophisticated problems in economic, engineering environment,... etc. It has been applied to several branches of mathematics such as operation research, game theory and among others. The soft set theory and it's applications increase after time to several researchers, especially in the recent years. This is because of the general nature of parameterizations expressed by a soft set. Therefore, due to these facts, several special sets have been introduced in the soft set theory and their properties have been studied within the soft topological space. The notion of topological spaces for soft sets was formulated by Shabir and Naz (2011), which is defined over an initial universe with a fixed set of parameters. Fuzzy soft set which is a combination of fuzzy and soft sets were first introduced by Maji, Biswas, and Roy (2001). Many researchers improved this study and gave new results (Ahmad & Kharal, 2009; Ali, 2011). Aygünoğlu and Aygün (2009) applied fuzzy soft sets on group theory. Tanay and Kandemir (2011) defined fuzzy soft topology on a fuzzy soft set over an initial universe. Clearly, the main reason for the inability to study the concept of fuzzifying soft topology is to disable the concept of a soft point to satisfy some important properties as it does not belong to either the soft set or to its complement. Additionally, the concept of distinct two soft points has not been defined yet. There is no doubt that the concept of soft element

Allam, Ismail, and Mohammed (2017) exceeded over these deficiencies and enable us to define and study the concept of fuzzifying soft topology as will be discussed.

It should be pointed out that the researchers Khalil (2015); Sayed and Borzooei (2016) who worked on this topic were not able to give a good account of. Mohammed, Ismail, and Allam (2018) used the soft element to redefine the notion of fuzzifying structure of soft set theory. In the present study we consider the concepts: soft semi open sets, soft semi interior sets, soft semi neighborhood system and soft semi closure operator in fuzzifying soft topologecal spaces. Furthermore, we introduce and study the soft semi continuity.

2. Preliminaries

In this section, we present the basic definitions and results of fuzzy logic, soft set theory and fuzzifying soft topology which will be needed in the sequel.

Definition 2.1. Zadeh (1965); Let X be a nonempty and $\Im(X)$ be the family of all functions from X into the closed unit interval $I=[0,1]$. Each member of $\Im(X)$ is called a fuzzy set of X. The ordinary inclusion relation "$\leq$ "on $\Im(X)$ and fuzzy set operations "$\sqcup$ ", "$\sqcap$ "and "$-$ "are defined as follows:

1. For any $\tilde{A}, \tilde{B} \in \Im(X), \tilde{A} \leq \tilde{B}$ if and only if $\tilde{A}(x) \leq \tilde{B}(x), \forall x \in X$;
2. For any $\{\tilde{A}_j : j \in J\} \sqsubseteq \Im(X), (\sqcup_{j\in J} \tilde{A}_j)(x) = \vee_{j\in J} \tilde{A}_j(x), \forall x \in X$;
3. For any $\{\tilde{A}_j : j \in J\} \sqsubseteq \Im(X), (\sqcap_{j\in J} \tilde{A}_j(x) = \wedge_{j\in J} \tilde{A}_j(x), \forall x \in X$;
4. The complement $(X\backslash\tilde{A}) \in \Im(X)$ of $\tilde{A} \in \Im(X)$ is defined as $(X\backslash\tilde{A})(x) = 1 - \tilde{A}(x)$ for every $x \in X$.

Definition 2.2. Ying (1991); Let $\tilde{A} \in \Im(X)$. Then $\tilde{A}$ is called normal if there exists $x \in X$ such that $\tilde{A}(x) = 1$.

Definition 2.3. In the following, we present the fuzzy logic and corresponding set-theoretical notations Ying (1991, 1992, 1993) For any formula φ, the symbol $[\varphi]$ means the truth value of φ, where the set of truth values is the unit interval $[0,1]$. A formula φ is valid, we write φ if and only if $[\varphi]=1$ for every interpretation. The truth valuation rules for primary fuzzy logical formulas and corresponding set theoretical notations are:

1. (a) $[\alpha] := \alpha(\alpha \in [0,1])$;

   (b) $[\varphi \wedge \psi] := \min([\varphi], [\psi])$;

   (c) $[\varphi \rightarrow \psi] := \min(1, 1 - [\varphi] + [\psi])$.
2. If $\tilde{A} \in \Im(X)$, then $[x \in \tilde{A}] := \tilde{A}(x)$.
3. If X is the universe of discourse, then $[\forall x \varphi(x)] := \inf_{x\in X}[\varphi(x)]$.

Definition 2.4. Ying (1991, 1992, 1993); The truth valuation rules for some derived formulae are:

1. $[\neg\varphi] := [\varphi \to 0] = 1 - [\varphi]$;

2. $[\varphi \vee \psi] := [(\neg(\neg\varphi \wedge \neg\psi)] = max([\varphi], [\psi])$;

3. $[\varphi \leftrightarrow \psi] := [(\varphi \to \psi) \wedge (\psi \to \varphi)]$;

4. $[\varphi \wedge \psi] := [\neg(\varphi \to \neg\psi)] = \max(0, [\varphi] + [\psi] - 1)$;

5. $[\varphi \dot{\vee} \psi] := [\neg\varphi \to \psi] = \min(1, [\varphi] + [\psi])$;

6. $[\exists x\varphi(x)] := [\neg\forall x \neg\varphi(x)] = \sup_{x\in X}[\varphi(x)]$;

7. If $\tilde{A}, \tilde{B} \in \Im(X)$, then

(a) $[\tilde{A} \sqsubseteq \tilde{B}] := [\forall x(x \in \tilde{A} \to x \in \tilde{B})] = \inf_{x\in X}\min(1, 1 - \tilde{A}(x) + \tilde{B}(x))$;

(b) $[\tilde{A} \equiv \tilde{B}] := [(\tilde{A} \sqsubseteq \tilde{B}) \wedge (\tilde{B} \sqsubseteq \tilde{A})]$;

(c) $[\tilde{A} \doteqdot \tilde{B}] := [(\tilde{A} \sqsubseteq \tilde{B}) \wedge (\tilde{B} \sqsubseteq \tilde{A})]$.

Definition 2.5. Molodtsov (1999) Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F, A) denoted by F_A is called a soft set over X, where F is a mapping given by $F: A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F, A) and if $e \notin A$, then $F(e) = \Phi$ *i.e* $F_A = \{F(e): e \in A \sqsubseteq E, F: A \to P(X)\}$. The family of all these soft sets over X denoted by $SS(X)_A$.

Definition 2.6. Maji, Biswas, and Roy (2003); Let X be an initial universe and A be a set of parameters. Then,

1. A soft set (F, A) over X is said to be a NULL soft set, denoted by (Φ, A) or Φ_A, if $F(e) = \Phi$ for all $e \in A$.

2. A soft set (F, A) over X is said to be an absolute soft set, denoted by (X, A) or X_A, if $F(e) = X$ for all $e \in A$. Clearly we have $(\Phi, A)^c = (X, A)$ and $(X, A)^c = (\Phi, A)$.

Definition 2.7. Allam et al. (2017); A soft set (M, A) over X is called a soft element, denoted by e/x or e_x *or* e_M if $M(e) = \{x\}$, $M(e') = \Phi$, for all $e' \in A - \{e\}$, and $SE(X)$ denoted the set of all soft elements in X

1. $e_M \tilde{\in} (F, A)$ read as e_M belongs to the soft set (F, A) if $M(e) \tilde{\sqsubseteq} F(e)$;

2. e_{i_M} and e_{j_N} are two distinct soft elements if $x \neq y$.

Definition 2.8. P. Maji et al. (2003); The union of two soft sets (F, A) and (G, B) over the common universe X and E be a set of parameters is the soft set (H, C), where $C = A \sqcup B$ and $\forall e \in C$ we have

$$H(e) = \begin{cases} F(e) & If e \in A - B, \\ G(e) & if e \in B - A, \\ F(e) \sqcup G(e) & if e \in A \sqcap B \end{cases}$$

This relationship is written as $(H, C) = (F, A)\ \tilde{\sqcup}\ (G, B)$.

Definition 2.9. Maji et al. (2003); The intersection of two soft sets (F, A) *and* (G, B) over the common universe X is the soft set (H, C), where $C = A \sqcap B$ and for all $e \in C, H(e) = F(e) \sqcap G(e)$. This relationship is written as $(H, C) = (F, A) \sqcap (G, B)$.

Definition 2.10. Maji et al. (2001); A pair (f_A) or (f, A) is called a fuzzy soft set over X, where f is a mapping given by $f: A \to \mathfrak{I}(X)$. The set of all fuzzy soft sets over X, denoted by $\mathfrak{I}(SS(X))$.

Definition 2.11. Roy and Samanta (2012); Let $A \sqsubseteq E$. (f_A, E) is defined to be a fuzzy soft set on (X, E) if $f_A: E \to \mathfrak{I}(X)$ is a mapping defined by $f_A(e) = \mu_{f_A}^e$ where $\mu_{f_A}^e = \bar{\Phi}$ if $e \in E - A$ and $\mu_{f_A}^e \neq \bar{\Phi}$ if $e \in A$.

Definition 2.12. Roy and Samanta (2012); The complement of fuzzy soft set (f_A, E) on (X, E) is a fuzzy soft set (f_A^c, E) which is denoted by $(f_A, E)^c$ and $f_A^c: E \to \mathfrak{I}(X)$ is defined by $\mu_{f_A^c}^e = 1 - \mu_{f_A}^e$ if $e \in A$ and $\mu_{f_A^c}^e = \bar{X}$ if $e \in E - A$.

Definition 2.13. Borgohain and Gohain (2014); A fuzzy soft set (f, A) is defined on X. The α-cut set ${}^{\alpha}(f, A)$ is made up of members X. Whose members grade is not less than α, therefore ${}^{\alpha}(f, A) = \{e_x^\delta | (f, A)(e_x^\delta) \geq \alpha\}$ Where $\alpha \in [0,1]$.

Definition 2.14. Khalil (2015), Sayed and Borzooei (2016); Let X be an initial universe, $\tilde{\tau} \in \mathfrak{I}(SS(X))$ satisfy the following conditions:

(1) $\tilde{\tau}(\Phi_A) = \tilde{\tau}(X_A) = 1$;

(2) For any $F_A, G_B\ \tilde{\in}\ SS(X), \tilde{\tau}(F_A\ \widetilde{\sqcap\ \acute{o}}G_B) \geq \tilde{\tau}(F_A) \wedge \tilde{\tau}(G_B)$;

(3) For any $\{(F_A)_\lambda : \lambda \in \Lambda\}\ \tilde{\sqsubseteq}\ SS(X), \tilde{\tau}(\tilde{\sqcup}_{\lambda \in \Lambda} (F_A)_\lambda) \geq \bigwedge_{\lambda \in \Lambda} \tilde{\tau}(F_A)_\lambda$.

Where $\Phi_A = 0_A$ and $X_A = 1_A$ then, $\tilde{\tau}$ is a fuzzifying soft topology and $(X, \tilde{\tau}, A)$ is a fuzzifying soft topological space.

Definition 2.15. Khalil (2015), Sayed and Borzooei (2016); Let $(X, \tilde{\tau}, A)$ be a fuzzifying soft topological space and $F_A\ \tilde{\in}\ SS(X)$. The family of all fuzzifying soft closed sets, denoted by $\tilde{\mathcal{F}} \in \mathfrak{I}(SS(X))$, is defined as $F_A\ \tilde{\in}\ \tilde{\mathcal{F}} := (X_A \backslash F_A)\ \tilde{\in}\ \tilde{\tau}$, where $X_A \backslash F_A$ is the complement of F_A.

Definition 2.16. Mohammed et al. (2018); Let $(X, \tilde{\tau}, A)$ be a fuzzifying soft topological space and $F_A\ \tilde{\in}\ SS(X_A)$. The fuzzifying soft closure of a soft set F_A is denoted by $Cl(F_A)$ and defined as follows;

$e_M\ \tilde{\in}\ Cl(F_A) := \forall G_B((F_A\ \tilde{\sqsubseteq}\ \mathrm{G}_B) \wedge (G_B\ \tilde{\in}\tilde{\sqsubseteq}) \to e_M\ \tilde{\in}\ G_B)$,

i.e., $Cl(F_A)(e_M) = \inf_{e_M \tilde{\notin} G_B \tilde{\sqsupseteq} F_A} \left(1 - \tilde{\sqsubseteq}\ (G_B)\right)$.

Definition 2.17. Mohammed et al. (2018); Let $(X, \tilde{\tau}, A)$ be a fuzzifying soft topological space and $F_A\ \tilde{\in}\ SS(X_A)$. The fuzzifying soft interior of a soft set F_A is denoted by $Int(F_A)$ and defined as follows;

$e_M \tilde{\in} Int(F_A) := F_A \tilde{\in} N_{e_M}$, i.e., $Int(F_A)(e_M) = N_{e_M}(F_A)$.

Definition 2.18. Mohammed et al. (2018); Let $Cl: SS(X_A) \to \Im(SS(X_A))$. If its extension $Cl: \Im(SS(X_A)) \to \Im(SS(X_A))$, defined by

$Cl(\tilde{F}_A) = \sqcup_{\alpha \in [0,1]} \alpha Cl(\tilde{F}_A)_\alpha$, $\tilde{F}_A \tilde{\in} \Im(SS(X_A))$, (Where $\tilde{F}_{A_\alpha} = \{e_x^\delta | \tilde{F}_A(e_x^\delta) \geq \alpha\}$ is the soft α −cut of $\tilde{F}_A$ and $\alpha Cl \tilde{F}_A = \alpha \wedge \tilde{F}_A(e_x^\delta)$) satisfies the following Kuratowski closure axioms:

(1) $Cl(\Phi_A) = \Phi_A$;

(2) $\tilde{F}_A \tilde{\sqsubseteq} Cl(\tilde{F}_A), \tilde{F}_A \tilde{\in} \Im(SS(X_A))$;

(3) $Cl(\tilde{F}_A \tilde{\sqcup} \tilde{G}_A) = Cl(\tilde{F}_A) \tilde{\sqcup} Cl(\tilde{G}_A), \tilde{F}_A, \tilde{G}_A \tilde{\in} \Im(SS(X_A))$;

(4) $Cl(Cl(\tilde{F}_A)) \tilde{\sqsubseteq} Cl(\tilde{F}_A), \tilde{F}_A \tilde{\in} \Im(SS(X_A))$.

Then $Cl: SS(X_A) \to \Im(SS(X_A))$ is called a fuzzifying soft closure operator.

Theorem 2.19. Mohammed et al. (2018); Let $(X, \tilde{\tau}, A)$ be a fuzzifying soft topological space. Then

(1) $\models Int(F_A) \tilde{\sqsubseteq} Int(Int(F_A))$, for any $(F_A) \tilde{\in} SS(X_A)$

(2) $\models Int(F_A \tilde{\sqcap} G_A) \equiv Int(F_A) \tilde{\sqcap} Int(G_A)$, for any $F_A, G_A \tilde{\in} SS(X_A)$.

(3) $\models (X_A \backslash Cl(Int(F_A))) \equiv Int(Cl(X_A \backslash F_A))$, for any $(F_A) \tilde{\in} SS(X_A)$

(4) $\models (X_A \backslash Int(Cl(F_A))) \equiv Cl(Int(X_A \backslash F_A))$, for any $(F_A) \tilde{\in} SS(X_A)$

(5) $\models F_A \tilde{\sqsubseteq} G_A \to Cl(Int(F_A)) \tilde{\sqsubseteq} Cl(Int(G_A))$, where $F_A, G_A \tilde{\in} SS(X_A)$.

3. Basic Properties On Soft Semi-Open Sets in Fuzzifying Soft Topological Spaces

In this section we introduce and study the concepts of fuzzifying soft semi-neighborhood system, fuzzifying soft semi-derived, fuzzifying soft semi-closure and fuzzifying soft semi-interior of soft set. Finally the relationship between these notions are established and supporte them by some examples.

Definition 3.1. Let $(X, \tilde{\tau}, A)$ be a fuzzifying soft topological space.
(1) The family of all fuzzifying soft semi-open sets is denoted by
$S\tilde{\tau} \tilde{\in} \Im(SS(X_A))$ and defined as follows;
$F_A \tilde{\in} S\tilde{\tau} := \forall e_M (e_M \tilde{\in} F_A \to e_M \tilde{\in} Cl(Int(F_A)))$. i.e., $S\tilde{\tau}(F_A) = \inf_{e_M \tilde{\in} F_A} Cl(Int(F_A))(e_M)$;
(2) The family of all fuzzifying soft semi-closed sets is denoted by $S\tilde{\mathcal{F}}$ and defined as follows;
$F_A \tilde{\in} S\tilde{\mathcal{F}} := X_A \backslash F_A \tilde{\in} S\tilde{\tau}$.

Theorem 3.2. Let $(X, \tilde{\tau}, A)$ be a fuzzifying soft topological space. Then
(1) $S\tilde{\tau}(X_A) = 1, S\tilde{\tau}(\Phi_A) = 1$;
(2) for any $\{(F_A)_\lambda : \lambda \in \Lambda\}, S\tilde{\tau}(\sqcup_{\lambda \in \Lambda} (F_A)_\lambda) \geq \wedge_{\lambda \in \Lambda} S\tilde{\tau}(F_A)_\lambda$;
Proof. (1) It is straightforward.

(2) From Theorem 2.19, $\models Cl(Int(F_A)_\lambda) \widetilde{\sqsubseteq} Cl(Int(\widetilde{\sqcup}_{\lambda\in\Lambda} (F_A)_\lambda))$.

So $S\tilde{\tau}(\widetilde{\sqcup}_{\lambda\in\Lambda} (F_A)_\lambda) = \inf_{e_M \widetilde{\in} \widetilde{\sqcup}_{\lambda\in\Lambda}(F_A)_\lambda} Cl(Int(\widetilde{\sqcup}_{\lambda\in\Lambda} (F_A)_\lambda))(e_M) = \inf_{\lambda\in\Lambda} \inf_{e_M \widetilde{\in} (F_A)_\lambda} Cl(Int$
$(\widetilde{\sqcup}_{\lambda\in\Lambda} (F_A)_\lambda))(e_M) \geq \inf_{\lambda\in\Lambda} \inf_{e_M \widetilde{\in} (F_A)_\lambda} Cl(Int(F_A)_\lambda)(e_M) = \bigwedge_{\lambda\in\Lambda} S\tilde{\tau}(F_A)_\lambda$

Theorem 3.3. Let$(X,\tilde{\tau},A)$ be a fuzzifying soft topological space. Then
(1) $S\widetilde{\mathcal{F}}(X_A) = 1, S\widetilde{\mathcal{F}}(\Phi_A) = 1$;
(2) For any $\{F_{A_\lambda}: \lambda \in \Lambda\}, S\widetilde{\mathcal{F}}(\widetilde{\sqcap}_{\lambda\in\Lambda} F_{A_\lambda}) \geq \bigwedge_{\lambda\in\Lambda} S\widetilde{\mathcal{F}}(F_{A_\lambda})$.
Proof. follows; from Theorem 3.2.

Theorem 3.4. Let$(X,\tilde{\tau},A)$ be a fuzzifying soft topological space. Then
(1) $\models F_A \widetilde{\in} \tilde{\tau} \to F_A \widetilde{\in} S\tilde{\tau}$;
(2) $\models F_A \widetilde{\in} \widetilde{\mathcal{F}} \to F_A \widetilde{\in} S\widetilde{\mathcal{F}}$.
Proof. From the properties of interior and closure operator in the Definitions 2.16,2.17 and Theorem 2.19 we have
(1) $[F_A \widetilde{\in} \tilde{\tau}] = [F_A \widetilde{\sqsubseteq} Int(F_A)] \leq [F_A \widetilde{\sqsubseteq} ClInt(F_A)] = [F_A \widetilde{\in} S\tilde{\tau}]$.
(2) Follows from from (1)
The converse of Theorem3.4 generally need not be true as shown by the following example:

Example 3.5. Let $X = \{x_1, x_2\}$, $A = \{e_1, e_2\}$, and $\tilde{\tau} \widetilde{\in} \Im(SS(X_A))$ be a fuzzifying soft topology defined as follows;

$$\tilde{\tau}(F_A) = \begin{cases} 1 & \text{if} \quad F_A \widetilde{\in} \{\Phi_A, X_A\}, \\ \frac{1}{4} & \text{if} \quad F_A \widetilde{\in} \{\{(e_1,\Phi),(e_2,X)\},\{(e_1,\{x_2\}),(e_2,X)\}\}, \\ \frac{3}{4} & \text{if} \quad F_A = \{(e_1,\{x_2\}),(e_2,\Phi)\}, \\ 0 & \text{otherwise.} \end{cases}$$

For $H_A = \{(e_1,\{x_2\}),(e_2,\{x_2\})\}$, $IntH_A(e_1/x_1) = \sup_{e_1/x_1 \widetilde{\in} F_A \widetilde{\sqsubseteq} H_A} \tilde{\tau}(F_A) = 0$, $IntH_A(e_1/x_2) = \sup_{e_1/x_2 \widetilde{\in} F_A \widetilde{\sqsubseteq} H_A} \tilde{\tau}(F_A) = \frac{3}{4}$, $IntH_A(e_2/x_1) = \sup_{e_2/x_1 \widetilde{\in} F_A \widetilde{\sqsubseteq} H_A} \tilde{\tau}(F_A) = 0$ and $IntH_A(e_2/x_2) = \sup_{e_2/x_2 \widetilde{\in} F_A \widetilde{\sqsubseteq} H_A} \tilde{\tau}(F_A) = 0$. Hence $Int(H_A) = \tilde{G}_A = (\{(e_1,\{x_1/0, x_2/\frac{3}{4}\}),(e_2,\{x_1/0, x_2/0\})\})$, which is a fuzzy soft set. $(\tilde{G}_A)_{\frac{3}{4}} = \{(e_1,\{x_2\}),(e_2,\Phi)\}$, $(\tilde{G}_A)_0 = \{(e_1,\{x_1,x_2\}),(e_2,\{x_1,x_2\})\}$ and $Cl(\tilde{G}_A)_{\frac{3}{4}}(e_1/x_1) = 1 - \sup_{e_1/x_1 \widetilde{\in} F_A \widetilde{\sqsubseteq} X_A \setminus (\tilde{G}_A)_{\frac{3}{4}}} (\tilde{\tau}(F_A)) = 1 - 0 = 1$. Similarly one can easily get $Cl(\tilde{G}_A)_{\frac{3}{4}} = \{(e_1,\{x_1/1, x_2/1\}),(e_2,\{x_1/\frac{3}{4}, x_2/\frac{3}{4}\})\}$, and $Cl(\tilde{G}_A)_0 = \{(e_1,\{x_1/1,x_2/1\}),(e_2,\{x_1/1,x_2/1\})\}$. Then $Cl(\tilde{G}_A)(e_1/x_1) = \sup\{(\frac{3}{4} \wedge 1),(1 \wedge 0)\} = \frac{3}{4}$, $Cl(\tilde{G}_A)(e_1/x_2) = \frac{3}{4}$, $Cl(\tilde{G}_A)(e_2/x_1) = \frac{3}{4}$, $Cl(\tilde{G}_A)(e_2/x_2) = \frac{3}{4}$. Therefore $Cl(\tilde{G}_A) = \{(e_1,\{x_1/\frac{3}{4}, x_2/\frac{3}{4}\}),(e_2,\{x_1/\frac{3}{4}, x_2/\frac{3}{4}\})\} = Cl(Int(H_A))$ and hence $S\tilde{\tau}(H_A) = \inf_{e/x \widetilde{\in} H_A} Cl(Int(H_A)) = \frac{3}{4} \nleq \tilde{\tau}(H_A) = 0$.

Theorem 3.6. Let $(X,\tilde{\tau},A)$ be a fuzzifying soft topological space and $(F_A) \widetilde{\in} SS(X_A)$. Then

(1) $\models Cl(F_A) \equiv ClInt(F_A) \leftrightarrow F_A \tilde{\in} S\tilde{\tau}$;
(2) $\models Int(F_A) \equiv IntCl(F_A) \leftrightarrow F_A \tilde{\in} S\tilde{\mathcal{F}}$.
Proof. (1)$[Cl(F_A) \equiv ClInt(F_A)] = [Cl(F_A) \tilde{\sqsubseteq} ClInt(F_A)] \wedge [ClInt(F_A) \tilde{\sqsubseteq} Cl(F_A)]$.
So $[Cl(F_A) \tilde{\sqsubseteq} ClInt(F_A)] \leq [(F_A) \tilde{\sqsubseteq} ClInt(F_A)] = [(F_A) \tilde{\in} S\tilde{\tau}]$;
Conversely. $[F_A \tilde{\in} S\tilde{\tau}] = [F_A \tilde{\sqsubseteq} ClInt(F_A)] \leq [Cl(F_A) \tilde{\sqsubseteq} Cl(Cl(Int(F_A)))]$ we have $[Cl(Cl(Int(F_A))) \tilde{\sqsubseteq} Cl(Int(F_A))] = 1$. Since $[Int(F_A) \tilde{\sqsubseteq} (F_A)] = 1$, so $[ClInt(F_A) \tilde{\sqsubseteq} Cl(F_A)] = 1$, hence $[Cl(F_A) \tilde{\sqsubseteq} ClInt(F_A)] \wedge [ClInt(F_A) \tilde{\sqsubseteq} Cl(F_A)] = 1$. Therefore $[Cl(Int(F_A)) \equiv Cl(F_A)]$.
(2)Is similar to (1).

Theorem 3.7. Let $(X, \tilde{\tau}, A)$ be a fuzzifying soft topological space and $(F_A) \tilde{\in} SS(X_A)$. Then
(1) $\models F_A \tilde{\in} S\tilde{\tau} \leftrightarrow \forall e_M(e_M \tilde{\in} F_A \rightarrow \exists G_A(G_A \tilde{\in} S\tilde{\tau} \wedge e_M \tilde{\in} G_A \tilde{\sqsubseteq} F_A))$;
(2) $\models F_A \tilde{\in} S\tilde{\mathcal{F}} \leftrightarrow \forall e_M(e_M \tilde{\in} IntCl(F_A) \rightarrow e_M \tilde{\in} F_A)$;
Proof. (1) $[\forall e_M(e_M \tilde{\in} F_A \rightarrow \exists G_A(G_A \tilde{\in} S\tilde{\tau} \wedge e_M \tilde{\in} G_A \tilde{\sqsubseteq} F_A))] = \inf_{e_M \tilde{\in} F_A} \sup_{e_M \tilde{\in} G_A \tilde{\sqsubseteq} F_A} S\tilde{\tau}(G_A)$. It is clear that $\inf_{e_M \tilde{\in} F_A} S\tilde{\tau}(G_A) \geq S\tilde{\tau}(F_A)$. In the other hand, let $\mathfrak{B}_{e_M} = \{G_A : e_M \tilde{\in} G_A \tilde{\sqsubseteq} F_A\}$. Then, for any $f \in \prod_{e_M \tilde{\in} F_A} \mathfrak{B}_{e_M}$, we have $\bigsqcup_{e_M \tilde{\in} F_A} f(e_M) = F_A$ and so $S\tilde{\tau}(F_A) = S\tilde{\tau}(\bigsqcup_{e_M \tilde{\in} F_A} f(e_M)) \geq \inf_{e_M \tilde{\in} F_A} S\tilde{\tau}(f(e_M))$. Thus,

$$S\tilde{\tau}(F_A) \geq \sup_{f \in \prod_{e_M \tilde{\in} F_A} \mathfrak{B}_{e_M}} \inf_{e_M \tilde{\in} F_A} S\tilde{\tau}(f(e_M)) = \inf_{e_M \tilde{\in} F_A} \sup_{e_M \tilde{\in} G_A \tilde{\sqsubseteq} F_A} S\tilde{\tau}(G_A).$$

(2) $[\forall e_M(e_M \tilde{\in} IntCl(F_A) \rightarrow e_M \tilde{\in} F_A)] = \left[\forall e_M\left(e_M \tilde{\in} X_A \backslash F_A \rightarrow e_M \tilde{\in} X_A \backslash IntCl(F_A)\right)\right]$
$= \inf_{e_M \tilde{\in} X_A \backslash F_A} (X_A \backslash IntCl(F_A))(e_M) = \inf_{e_M \in X_A \backslash F_A} (ClInt(X_A \backslash F_A))(e_M) = [X_A \backslash F_A \tilde{\in} S\tilde{\tau}] = [F_A \tilde{\in} S\tilde{\mathcal{F}}]$.

Lemma 3.8. For any $F_A, H_A \tilde{\sqsubseteq} SS(X_A)$.
(1) $\models H_A \doteq Int(F_A) \rightarrow H_A \tilde{\sqsubseteq} F_A$;
(2) $\models H_A \doteq Int(F_A) \wedge F_A \tilde{\in} S\tilde{\tau} \rightarrow F_A \tilde{\sqsubseteq} Cl(H_A)$.
Proof. (1)$[H_A \doteq Int(F_A)] = [(H_A \tilde{\sqsubseteq} Int(F_A)) \wedge (Int(F_A) \tilde{\sqsubseteq} H_A)]$. If $[H_A \tilde{\sqsubseteq} F_A] = 0$, then $[H_A \tilde{\sqsubseteq} Int(F_A)] = 0$.
Therefore $[H_A \doteq Int(F_A)] = 0$.
(2)$[(H_A \doteq Int(F_A)) \wedge F_A \tilde{\in} S\tilde{\tau}] = [(H_A \doteq Int(F_A)) \wedge F_A \tilde{\sqsubseteq} Cl(Int(F_A))]$
$\leq [(Int(F_A) \tilde{\sqsubseteq} H_A) \wedge (F_A \tilde{\sqsubseteq} Cl(Int(F_A)))] \leq [Cl(Int(F_A) \tilde{\sqsubseteq} Cl(H_A)) \wedge (F_A \tilde{\sqsubseteq} Cl(Int(F_A)))] \leq [F_A \tilde{\sqsubseteq} Cl(H_A)]$.

Theorem 3.9. Let $(X, \tilde{\tau}, A)$ be a fuzzifying soft topological space. Then
(1) $\models \exists G_A(G_A \tilde{\in} \tilde{\tau} \wedge G_A \tilde{\sqsubseteq} F_A \tilde{\sqsubseteq} Cl(G_A)) \leftrightarrow F_A \tilde{\in} S\tilde{\tau}$;
(2) $\models \exists H_A(H_A \tilde{\in} \tilde{\mathcal{F}} \wedge Int(H_A) \tilde{\sqsubseteq} C_A \tilde{\sqsubseteq} H_A) \leftrightarrow C_A \tilde{\in} S\tilde{\mathcal{F}}$.
Proof. (1) $[\exists G_A(G_A \tilde{\in} \tilde{\tau} \wedge G_A \tilde{\sqsubseteq} F_A \tilde{\sqsubseteq} Cl(G_A))] = \sup_{G_A \tilde{\in} \mathbf{SS}(\mathbf{X_A})} ([G_A \tilde{\in} \tilde{\tau}] \wedge [G_A \tilde{\sqsubseteq} F_A] \wedge [F_A \tilde{\sqsubseteq} Cl(G_A)])$
$= \sup_{G_A \tilde{\sqsubseteq} F_A} ([G_A \tilde{\sqsubseteq} Int(G_A)] \wedge [G_A \tilde{\sqsubseteq} F_A] \wedge [F_A \tilde{\sqsubseteq} Cl(G_A)]) \leq \sup_{G_A \tilde{\sqsubseteq} F_A} ([G_A \tilde{\sqsubseteq} Int(G_A)] \wedge [Int(G_A)$

$\tilde{\sqsubseteq} Int(F_A)] \wedge [F_A \tilde{\sqsubseteq} Cl(G_A)]) \leq \sup_{G_A \tilde{\sqsubseteq} F_A} ([G_A \tilde{\sqsubseteq} Int(F_A)] \wedge [F_A \tilde{\sqsubseteq} Cl(G_A)])$

$\leq \sup_{G_A \tilde{\sqsubseteq} F_A} ([Cl(G_A) \tilde{\sqsubseteq} Cl(Int(F_A))] \wedge [F_A \tilde{\sqsubseteq} Cl(G_A))]) \leq \sup_{G_A \tilde{\sqsubseteq} F_A} [F_A \tilde{\sqsubseteq} Cl(Int(F_A))]$

$= [F_A \tilde{\in} S\tilde{\tau}]$.

Conversely $[F_A \tilde{\in} S\tilde{\tau}] = \left[F_A \tilde{\sqsubseteq} Cl\left(Int(F_A)\right)\right] = \left[Int(F_A) \tilde{\sqsubseteq} F_A \tilde{\sqsubseteq} Cl\left(Int(F_A)\right)\right]$

$\leq \sup_{G_A \tilde{\in} \tilde{\tau} \wedge G_A \tilde{\sqsubseteq} Int(F_A)} [G_A \tilde{\sqsubseteq} F_A \tilde{\sqsubseteq} Cl(G_A)] = \exists G_A(G_A \tilde{\in} \tilde{\tau} \wedge G_A \tilde{\sqsubseteq} F_A \tilde{\sqsubseteq} Cl(G_A))$.

(2)$[C_A \tilde{\in} S\tilde{\mathcal{F}}] = [X_A \backslash C_A \tilde{\in} S\tilde{\tau}] \geq [\exists G_A(G_A \tilde{\in} \tilde{\tau} \wedge G_A \tilde{\sqsubseteq} X_A \backslash C_A \tilde{\sqsubseteq} Cl(G_A))] = [\exists G_A(G_A \tilde{\in} \tilde{\tau} \wedge (X_A \backslash Cl(G_A) \tilde{\sqsubseteq} C_A \tilde{\sqsubseteq} X_A \backslash G_A))] = [\exists G_A(G_A \tilde{\in} \tilde{\tau} \wedge Int(X_A \backslash G_A) \tilde{\sqsubseteq} C_A \tilde{\sqsubseteq} X_A \backslash G_A))]$ put $X_A \backslash G_A = H_A$, $= [\exists H_A(H_A \tilde{\in} \tilde{\mathcal{F}} \wedge Int(H_A) \tilde{\sqsubseteq} C_A \tilde{\sqsubseteq} H_A)]$. Hence the proof.

Conversely

$[\exists H_A(H_A \tilde{\in} \tilde{\mathcal{F}} \wedge Int(H_A) \tilde{\sqsubseteq} C_A \tilde{\sqsubseteq} H_A)] = \sup_{H_A \tilde{\in} SS(X_A)} ([H_A \tilde{\in} \tilde{\mathcal{F}}] \wedge [Int(H_A) \tilde{\sqsubseteq} C_A] \wedge [C_A \tilde{\sqsubseteq} H_A]) =$

$\sup_{H_A \tilde{\in} SS(X_A)} ([Cl(H_A) \tilde{\sqsubseteq} H_A] \wedge [Int(H_A) \tilde{\sqsubseteq} C_A] \wedge [C_A \tilde{\sqsubseteq} H_A]) \leq \sup_{Int(H_A) \tilde{\sqsubseteq} C_A} ([Cl(H_A) \tilde{\sqsubseteq} H_A] \wedge$

$[Int(H_A) \tilde{\sqsubseteq} C_A] \wedge [C_A \tilde{\sqsubseteq} H_A]) \leq \sup_{Int(H_A) \tilde{\sqsubseteq} C_A} ([Cl(H_A) \tilde{\sqsubseteq} H_A] \wedge [Int(Cl(H_A)) \tilde{\sqsubseteq} C_A] \wedge [C_A \tilde{\sqsubseteq} H_A]) \leq$

$\sup_{Int(H_A) \tilde{\sqsubseteq} C_A} ([Int(Cl(H_A)) \tilde{\sqsubseteq} C_A] \wedge [C_A \tilde{\sqsubseteq} H_A]) \leq \sup_{Int(H_A) \tilde{\sqsubseteq} C_A} [Int(Cl(C_A)) \tilde{\sqsubseteq} C_A] = [C_A \tilde{\in} S\tilde{\mathcal{F}}]$.

Definition 3.10. Let $(X, \tilde{\tau}, A)$ be a fuzzifying soft topological space, $e_M \tilde{\in} SS(X_A)$ and $F_A, G_A \tilde{\in} SS(X_A)$. The fuzzifying soft semi-neighborhood system of e_M, denoted by $SN_{e_M} \tilde{\in} \Im(SS(X_A))$ is defined as $F_A \tilde{\in} SN_{e_M} := \exists G_A((G_A \tilde{\in} S\tilde{\tau}) \wedge (e_M \tilde{\in} G_A \tilde{\sqsubseteq} F_A))$,
i.e., $SN_{e_M}(F_A) = \sup_{e_M \tilde{\in} G_A \tilde{\sqsubseteq} F_A} S\tilde{\tau}(G_A)$.

Example 3.11. Let $X = \{a, b\}$, $A = \{e_1, e_2\}$, and $\tilde{\tau} \tilde{\in} \Im(SS(X_A))$ be a fuzzifying soft topology defined as follows;

$$\tilde{\tau}(F_A) = \begin{cases} 1 & \text{if} \quad F_A \tilde{\in} \{\Phi_A, X_A\}, \\ \frac{3}{4} & \text{if} \quad F_A = \{(e_1, \Phi), (e_2, X)\}, \\ 0 & \text{otherwise.} \end{cases}$$

For a soft set $H_A = \{(e_1, \{a\}), (e_2, \{a, b\})\}$, $IntH_A(e_1/a) = \sup_{e_1/a \tilde{\in} F_A \tilde{\sqsubseteq} H_A} \tilde{\tau}(F_A) = 0$,
$IntH_A(e_1/b) = \sup_{e_1/b \tilde{\in} F_A \tilde{\sqsubseteq} H_A} \tilde{\tau}(F_A) = 0$, $IntH_A(e_2/a) = \sup_{e_2/a \tilde{\in} F_A \tilde{\sqsubseteq} H_A} \tilde{\tau}(F_A) = \frac{3}{4}$ and $IntH_A(e_2/b) = \sup_{e_2/b \tilde{\in} F_A \tilde{\sqsubseteq} H_A} \tilde{\tau}(F_A) = \frac{3}{4}$. Hence $Int(H_A) = \tilde{G}_A = (\{(e_1, \{a/0, b/0\}), (e_2, \{a/\frac{3}{4}, b/\frac{3}{4}\})\})$, which is a fuzzy soft set. $(\tilde{G}_A)_{\frac{3}{4}} = \{(e_1, \Phi), (e_2, \{a, b\})\}$, $(\tilde{G}_A)_0 = \{(e_1, \{a, b\}), (e_2, \{a, b\})\}$, $Cl(\tilde{G}_A)_{\frac{3}{4}}(e_1/a) = 1 - \sup_{e_1/a \tilde{\in} F_A \tilde{\sqsubseteq} X_A \backslash (\tilde{G}_A)_{\frac{3}{4}}} (\tilde{\tau}(F_A)) = 1 - 0 = 1$.Similarly one can easily get

$Cl(\tilde{G}_A)_{\frac{3}{4}} = \{(e_1, \{a/1, b/1\}), (e_2, \{a/1, b/1\})\}$, and $Cl(\tilde{G}_A)_0 = \{(e_1, \{a/1, b/1\}), (e_2, \{a/1, b/1\})\}$.
Hence $Cl(\tilde{G}_A)(e_1/a) = \sup\{(\frac{3}{4} \wedge 1), (0 \wedge 1)\} = \frac{3}{4}$, $Cl(\tilde{G}_A)(e_1/b) = \frac{3}{4}$, $Cl(\tilde{G}_A)(e_2/a) = \frac{3}{4}$, $Cl(\tilde{G}_A)(e_2/b) = \frac{3}{4}$.

Therefore $Cl(\tilde{G}_A) = \{(e_1, \{a/\frac{3}{4}, b/\frac{3}{4}\}), (e_2, \{a/\frac{3}{4}, b/\frac{3}{4}\})\} = Cl(Int(\mathrm{H}_A))$ and hence
$S\tilde{\tau}(H_A) = \inf_{e/x \tilde{\in} H_A} Cl(Int(H_A)) = \frac{3}{4}$. By the same way we can find the family of all fuzzifying soft

semi open sets as follows;

$$S\tilde{\tau}(F_A) = \begin{cases} 1 & \text{if} \quad F_A \tilde{\in} \{\Phi_A, X_A\}, \\ \frac{3}{4} & \text{if} \quad F_A \tilde{\in} \{\{(e_1,\Phi),(e_2,X)\},\{(e_1,\{a\}),(e_2,X)\},\{(e_1,\{b\}),(e_2,X)\}\} \\ 0 & \text{otherwise.} \end{cases}$$

For a soft element $e_M = e_1/b$ and soft set $F_{1_A} = \{(e_1,\{b\}),(e_2,\Phi)\}$, $F_{2_A} = \{(e_1,\{b\}),(e_2,\{b\})\}$, $F_{3_A} = \{(e_1,\{b\}),(e_2,X)\}$, $F_{4_A} = \{(e_1,X),(e_2,\Phi)\}$ and $F_{5_A} = X_A$, we have $SN_{e_M}(F_{1_A}) = \sup_{e_M \tilde{\in} G_B \tilde{\sqsubseteq} F_{1_A}} S\tilde{\tau}(G_B) = 0$, similarly $SN_{e_M}(F_{2_A}) = 0$, $SN_{e_M}(F_{3_A}) = \frac{3}{4}$, $SN_{e_M}(F_{4_A}) = 0$, and $SN_{e_M}(F_{5_A}) = 1$.

Theorem 3.12. Let $(X, \tilde{\tau}, A)$ be a fuzzifying soft topological space and $F_A, G_A, H_A \tilde{\in} SS(X_A)$. Then
(1) $\models F_A \tilde{\in} S\tilde{\tau} \leftrightarrow \forall e_M(e_M \tilde{\in} F_A \to \exists G_A(G_A \tilde{\in} SN_{e_M} \wedge G_A \tilde{\sqsubseteq} F_A))$.
(2) $\models N_{e_M}(F_A) \leq SN_{e_M}(F_A)$

Proof. (1)From Theorem 3.7 we have, $[\forall e_M(e_M \tilde{\in} F_A \to \exists G_A(G_A \tilde{\in} SN_{e_M} \wedge G_A \tilde{\sqsubseteq} F_A))]$
$= \inf_{e_M \tilde{\in} F_A} \sup_{G_A \tilde{\sqsubseteq} F_A} SN_{e_M}(G_A) = \inf_{e_M \tilde{\in} F_A} \sup_{G_A \tilde{\sqsubseteq} F_A} \sup_{e_M \tilde{\in} H_A \tilde{\sqsubseteq} G_A} S\tilde{\tau}(H_A) = \inf_{e_M \tilde{\in} F_A} \sup_{e_M \tilde{\in} H_A \tilde{\sqsubseteq} G_A} S\tilde{\tau}(H_A) = [F_A \tilde{\in} S\tilde{\tau}]$.
(2)From (1) of Theorem3.4 and Lemma3.8, we have $SN_{e_M}(F_A) = \sup_{e_M \tilde{\in} G_A \tilde{\sqsubseteq} F_A} S\tilde{\tau}(G_A) \geq \sup_{e_M \tilde{\in} G_A \tilde{\sqsubseteq} F_A} \tilde{\tau}(G_A) = N_{e_M}(F_A)$.

Corollary 3.13. $inf_{e_M \tilde{\in} F_A} SN_{e_M}(F_A) = S\tilde{\tau}(F_A)$.

Proof.The proof is obtained from Definition3.10 and Theorem3.12.

Theorem 2.14. The mapping $SN: S(X) \to \mathfrak{I}_E^N(SS(X_A))$, $e_F \mapsto SN_{e_M}$ where $\mathfrak{I}_E^N(SS(X_A))$ is the set of all normal fuzzy soft subsets of $SS(X_A)$ has the following properties:
(1) For any $e_M, F_A, \models F_A \tilde{\in} SN_{e_M} \to e_M \tilde{\in} F_A$;
(2) For any $e_M, F_A, G_A, \models F_A \tilde{\sqsubseteq} G_A \to (F_A \tilde{\in} SN_{e_M} \to G_A \tilde{\in} SN_{e_M})$;
(3) For any $e_M, F_A, \models F_A \tilde{\in} SN_{e_M} \to \exists H_A(H_A \tilde{\in} SN_{e_M} \wedge H_A \tilde{\sqsubseteq} F_A \wedge \forall e_H(e_H \tilde{\in} H_A \to H_A \tilde{\in} SN_{e_H}))$.

Proof.
(1) If $[SN_{e_M}(F_A) = 0]$, then the result holds. If $[F_A \tilde{\in} SN_{e_M}] = \sup_{e_M \tilde{\in} G_A \tilde{\sqsubseteq} F_A} S\tilde{\tau}(G_A) > 0$, then there exists G_A such that $e_M \tilde{\in} G_A \tilde{\sqsubseteq} F_A$. Now we have $[e_M \tilde{\in} F_A]$=1. Therefore $[F_A \tilde{\in} SN_{e_M}] \leq [e_M \tilde{\in} F_A]$.
(2) If $[F_A \tilde{\sqsubseteq} G_A] = 0$, then the result holds. If $[F_A \tilde{\sqsubseteq} G_A] = 1$, then
$SN_{e_M}(G_A) = \sup_{e_M \tilde{\in} H_A \tilde{\sqsubseteq} G_A} S\tilde{\tau}(H_A) \geq \sup_{e_M \tilde{\in} H_A \tilde{\sqsubseteq} F_A} S\tilde{\tau}(H_A) = SN_{e_M}(F_A)$.
(3) $[\exists H_A(H_A \tilde{\in} SN_{e_M} \wedge H_A \tilde{\sqsubseteq} F_A \wedge \forall e_H(e_H \tilde{\in} H_A \to H_A \tilde{\in} SN_{e_H}))] = \sup_{H_A \tilde{\sqsubseteq} F_A} (SN_{e_M}(H_A) \wedge \inf_{e_H \tilde{\in} H_A} (SN_{e_H}(H_A)))$
$= \sup_{H_A \tilde{\sqsubseteq} F_A} (SN_{e_M}(H_A) \wedge S\tilde{\tau}(H_A)) = \sup_{H_A \tilde{\sqsubseteq} F_A} (S\tilde{\tau}(H_A)) \geq \sup_{e_M \tilde{\in} H_A \tilde{\sqsubseteq} F_A} S\tilde{\tau}(H_A) = SN_{e_M} H_A = [F_A \tilde{\in} SN_{e_M}]$.

Definition 3.15. Let $(X, \tilde{\tau}, A)$ be a fuzzifying soft topological space and $F_A, G_A \tilde{\in} SS(X_A)$, the fuzzifying soft semi-derived set of a soft set F_A is denoted by $semi - d(F_A)$ and defined as follows;

$$e_M \tilde{\in} \text{semi}-d(F_A) := \forall G_A(G_A \tilde{\in} SN_{e_M} \to (G_A \tilde{\sqcap} (F_A\backslash\{e_M\}) \neq \Phi_A)),$$

i.e., $\text{semi}-d(F_A)(e_M) = \inf_{G_A\tilde{\sqcap}(F_A\backslash\{e_M\})=\Phi_A}(1-SN_{e_M}(G_A))$.

Example 3.16. Let us consider the fuzzifying soft toplogical space $(X,\tilde{\tau},A)$ in Example *3.11*. For a soft set $H_A = \{(e_1,X),(e_2,\Phi)\}$ we have $semi-d(H_A)(e_1/a) = \inf_{G_A\tilde{\delta}(H_A\backslash\{e_1/a\})=\Phi_A}(1-SN_{e_1/a}(G_A)) = \frac{1}{4}$. Similarly $semi-d(H_A)(e_1/b) = \frac{1}{4}$, $semi-d(H_A)(e_2/a) = \frac{1}{4}$ and $semi-d(H_A)(e_2/b) = \frac{1}{4}$. Hence $semi-d(H_A) = \{(e_1,\{a/\frac{1}{4},b/\frac{1}{4}\}),(e_2,\{a/\frac{1}{4},b/\frac{1}{4}\})\}$ which is a fuzzy soft set.

Lemma 3.17. Let$(X,\tilde{\tau},A)$ be a fuzzifying soft topological space and $F_A, G_A \tilde{\in} SS(X_A)$. Then

$$\text{semi}-d(F_A)(e_M) = 1-SN_{e_M}((X_A\backslash F_A)\tilde{\sqcup}\{e_M\});$$

Proof. $\text{semi}-d(F_A)(e_M) = \inf_{G_A\tilde{\sqcap}(F_A\backslash\{e_M\})=\Phi_A}(1-SN_{e_M}(G_A)) = 1-\sup_{G_A\tilde{\sqcap}(F_A\backslash\{e_M\})=\Phi_A} SN_{e_M}(G_A)$ $=1-\sup_{G_A\tilde{\sqsubseteq}(X_A\backslash F_A)\tilde{\sqcup}\{e_M\}} SN_{e_M}(G_A) = 1-SN_{e_M}((X_A\backslash F_A)\tilde{\sqcup}\{e_M\})$.

Theorem 3.18. Let$(X,\tilde{\tau},A)$ be a fuzzifying soft topological space and $F_A, G_A \tilde{\in} SS(X_A)$. Then

(1) $\models \text{semi}-d(\Phi_A) \equiv \Phi_A$;

(2) $\models F_A \tilde{\sqsubseteq} G_A \to \text{semi}-d(F_A) \tilde{\sqsubseteq} \text{semi}-d(G_A)$;

(3) $\models F_A \tilde{\in} S \tilde{\sqsubseteq}\leftrightarrow \text{semi}-d(F_A) \tilde{\sqsubseteq} F_A$.;

(4) $\text{semi}-d(F_A) \tilde{\sqsubseteq} d(F_A)$.

Proof. (1) From Lemma 3.17, we have

$$\begin{aligned}\text{semi}-d(\Phi_A)(e_M) &= 1-SN_{e_M}\big((X_A\backslash\Phi_A)\tilde{\sqcup}\{e_M\}\big)\\ &= 1-SN_{e_M}(X_A) = 1-1 = 0.\end{aligned}$$

(2) Let $F_A \tilde{\sqsubseteq} G_A$, then from Lemma 3.17 and part (2) of Theorem3.14, we have

$$\begin{aligned}\text{semi}-d(F_A)(e_M) &= 1-SN_{e_M}((X_A\backslash F_A)\tilde{\sqcup}\{e_M\})\\ &\leq 1-SN_{e_M}((X_A\backslash G_A)\tilde{\sqcup}\{e_M\})\\ &= \text{semi}-d(G_A)(e_M).\end{aligned}$$

(3) From Lemma 3.17 and part (1) of Theorem 3.12, we have $[\text{semi}-d(F_A) \tilde{\sqsubseteq} F_A] = \forall e_M(e_M \tilde{\in} \text{semi}-d(F_A) \to e_M \tilde{\in} F_A)$ $= \inf_{e_M\tilde{\in}X_A}\min(1,1-\text{semi}-d(F_A)(e_M)+[e_M \tilde{\in} F_A])$ $= \inf_{e_M\tilde{\in}X_A\backslash F_A}(1-\text{semi}-d(F_A)(e_M))$ $= \inf_{e_M\tilde{\in}X_A\backslash F_A}(1-(1-SN_{e_M}((X_A\backslash F_A)\tilde{\sqcup}\{e_M\})))$ $= \inf_{e_M\tilde{\in}X_A\backslash F_A} SN_{e_M}((X_A\backslash F_A)\tilde{\sqcup}\{e_M\})$

$= \inf_{e_M\tilde{\in}X_A\backslash F_A} SN_{e_M}(X_A\backslash F_A) = \inf_{e_M\tilde{\in}X_A\backslash F_A}\sup_{e_M\tilde{\in}G_A\tilde{\sqsubseteq}X_A\backslash F_A} S\tilde{\tau}(G_A) = S\tilde{\tau}(X_A\backslash F_A) = [F_A \tilde{\in} S\tilde{\boldsymbol{\mathcal{F}}}] = [S\tilde{\boldsymbol{\mathcal{F}}}(F_A)]$.

(4) From part (2) of Theorem 3.12 and Lemma 3.8, we have

$$\begin{aligned}\text{semi}-d(F_A)(e_M) &= 1-SN_{e_M}((X_A\backslash F_A)\tilde{\sqcup}\{e_M\})\\ &\leq 1-N_{e_M}((X_A\backslash F_A)\tilde{\sqcup}\{e_M\}) = d(F_A)(e_M).\end{aligned}$$

The converse of (4) of Theorem3.18 generally need not be true as shown by the following example:

Example 3.19. Let us consider the fuzzifying soft toplogical space $(X,\tilde{\tau},A)$ in Example 3.11. For the soft set $F_A = \{(e_1,\{a\}),(e_2,\Phi)\}$ we have $d(F_A)(e_1/b) = 1-N_{e_1/b}((X_A\backslash F_A)\tilde{\sqcup}\{e_1/b\}) = 1-0 = 1$ and $semi-d(F_A)\left(\frac{e_1}{b}\right) = 1-SN_{\frac{e_1}{b}}\left((X_A\backslash F_A)\tilde{\sqcup}\left\{\frac{e_1}{b}\right\}\right) = 1-\frac{3}{4} = \frac{1}{4}$ and hence, $d(F_A)\left(\frac{e_1}{b}\right) = 1 \not\leq$

$semi - d(F_A)\left(\frac{e_1}{b}\right) = \frac{1}{4}$.

Definition 3.20. Let $(X, \tilde{\tau}, A)$ be a fuzzifying soft topological space. The fuzzifying soft semi-closure of a soft set F_A is denoted by $semi - Cl(F_A)$ and defined as follows;
$e_M \tilde{\in} semi - Cl(F_A) := \forall G_A((G_A \tilde{\supseteq} F_A) \wedge (G_A \tilde{\in} S\tilde{\mathcal{F}}) \rightarrow e_M \tilde{\in} G_A)$, $semi - Cl(F_A)(e_M) = \inf_{e_M \notin G_A \tilde{\supseteq} F_A} (1 - S\tilde{\mathcal{F}}(G_A))$.

Example 3.21. Let us consider the fuzzifying soft toplogical space $(X, \tilde{\tau}, A)$ in Example 3.11. For the soft set $H_A = \{(e_1, \{a\}), (e_2, \Phi)\}$ we have $semi - Cl(H_A)(e_1/a) = 1 - SN_{e_1/a}(X_A \backslash H_A) = 1 - 0 = 1$. Similarly $semi - Cl(H_A)(e_1/b) = \frac{1}{4}$, $semi - Cl(H_A)(e_2/a) = \frac{1}{4}$, $semi - Cl(H_A)(e_2/b) = \frac{1}{4}$. Hence $semi - Cl(H_A) = \{(e_1, \{a/1, b/\frac{1}{4}\}), (e_2, \{a/\frac{1}{4}, b/\frac{1}{4}\})\}$ which is a fuzzy soft set.

Theorem 3.22. Let $(X, \tilde{\tau}, A)$ be a fuzzifying soft topological space and $F_A \tilde{\in} SS(X_A)$. Then
(1) $\models semi - Cl(F_A)(e_M) = 1 - SN_{e_M}(X_A \backslash F_A)$;
(2) $\models semi - Cl(\Phi_A) \equiv \Phi_A$;
(3) $\models F_A \tilde{\sqsubseteq} semi - Cl(F_A)$;
Proof.
(1) $semi - Cl(F_A)(e_M) = \inf_{e_M \notin G_A \tilde{\supseteq} F_A} (1 - S\tilde{\mathcal{F}}(G_A)) = \inf_{e_M \tilde{\in} X_A \backslash G_A \tilde{\subseteq} X_A \backslash F_A} (1 - S\tilde{\tau}(X_A \backslash G_A))$
$=1 - \sup_{e_M \tilde{\in} X_A \backslash G_A \tilde{\subseteq} X_A \backslash F_A} S\tilde{\tau}(X_A \backslash G_A) = 1 - SN_{e_M}(X_A \backslash F_A)$.
(2) From (1) and since $semi - Cl(\Phi_A)(e_M) = 1 - SN_{e_M}(X_A \backslash \Phi_A) = 1 - SN_{e_M}(X_A) = 1 - 1 = 0$.
(3) It is clear that for any $F_A \tilde{\in} SS(X_A)$ and any $e_M \tilde{\in} X_A$, if $e_M \notin F_A$, then $SN_{e_M}(F_A) = 0$. If $e_M \tilde{\in} F_A$, then $semi - Cl(F_A)(e_M) = 1 - SN_{e_M}(X_A \backslash F_A) = 1 - 0 = 1$. Then $[F_A \tilde{\sqsubseteq} semi - Cl(F_A)] = 1$.

Lemma 3.23. If $(X, \tilde{\tau}, A)$ is a fuzzifying soft topological space, then
$[\tilde{G}_A \tilde{\sqsubseteq} F_A] = [\tilde{G}_A \tilde{\sqcup} F_A \tilde{\sqsubseteq} F_A]$, for any $F_A \tilde{\in} SS(X_A)$ and $\tilde{G}_A \tilde{\in} \Im(SS(X_A))$.

Theorem 3.24. Let $(X, \tilde{\tau}, A)$ be a fuzzifying soft topological space and $F_A \tilde{\in} SS(X_A)$. Then
(1) $\models e_M \tilde{\in} semi - Cl(F_A) \leftrightarrow \forall G_A(G_A \tilde{\in} SN_{e_M} \rightarrow F_A \tilde{\sqcap} G_A \neq \Phi_A)$;
(2) $\models semi - Cl(F_A) \equiv F_A \tilde{\sqcup} semi - d(F_A)$;
(3) $\models F_A \equiv semi - Cl(F_A) \leftrightarrow F_A \tilde{\in} S\tilde{\mathcal{F}}$
Proof.
(1) $[\forall G_A(G_A \tilde{\in} SN_{e_M} \rightarrow F_A \tilde{\sqcap} G_A \neq \Phi_A)] = \inf_{G_A \tilde{\subseteq} X_A \backslash F_A} (1 - SN_{e_M}(G_A)) = 1 - SN_{e_M}(X_A \backslash F_A)$
$= [e_M \tilde{\in} semi - Cl(F_A)]$.
(2) If $e_M \tilde{\in} F_A$, then the result holds. If $e_M \notin F_A$, then
$(F_A \tilde{\sqcup} semi - d(F_A))(e_M) = \max(F_A(e_M), semi - d(F_A)(e_M))$
$= \max(F_A(e_M), 1 - SN_{e_M}((X_A \backslash F_A) \tilde{\sqcup}$
$\{e_M\})) = 1 - SN_{e_M}((X_A \backslash F_A) \tilde{\sqcup} \{e_M\}) = 1 - SN_{e_M}(X_A \backslash F_A) = semi - Cl(F_A)(e_M)$.
(3) from (3) of Theorem 3.18, Lemma3.17 and (2) above we have, and since $[F_A \tilde{\sqsubseteq} F_A \tilde{\sqcup} semi - d(F_A)] = 1$ we have $S\tilde{\mathcal{F}}(F_A) = [semi - d(F_A) \tilde{\sqsubseteq} F_A] = [semi - d(F_A) \tilde{\sqcup} F_A \tilde{\sqsubseteq} F_A] = [semi - d(F_A) \tilde{\sqcup} F_A \tilde{\sqsubseteq} F_A] \wedge$
$[F_A \tilde{\sqsubseteq} semi - d(F_A) \tilde{\sqcup} F_A] = [semi - d(F_A) \tilde{\sqcup} F_A \equiv F_A] = [semi - Cl(F_A) \equiv F_A]$.

Theorem 3.25. Let $(X, \tilde{\tau}, A)$ be a fuzzifying soft topological space and $F_{1A}, F_{2A}, F_{3A} \tilde{\in} SS(X_A)$. Then
(1) $\models F_{2A} \dot{\equiv} semi - Cl(F_{1A}) \rightarrow F_{2A} \tilde{\in} S\tilde{\mathcal{F}}$.
(2) $\models semi - Cl(F_{1A}) \tilde{\sqsubseteq} Cl(F_{1A})$;
(3) $\models F_{1A} \tilde{\sqsubseteq} F_{2A} \longrightarrow semi - Cl(F_{1A}) \tilde{\sqsubseteq} semi - Cl(F_{2A})$;

Proof.

(1) $[F_{1A} \tilde{\sqsubseteq} F_{2A}] = 0$, then $[F_{2A} \dot{\equiv} semi - Cl(F_{1A})] = 0$. Now, we suppose $[F_{1A} \tilde{\sqsubseteq} F_{2A}] = 1$ then $[F_{2A} \tilde{\sqsubseteq} semi - Cl(F_{1A})] = 1 - \sup_{e_M \tilde{\in} F_{2A} \backslash F_{1A}} SN_{e_M}(X_A \backslash (F_{1A}))$, $[semi - Cl(F_{1A}) \tilde{\sqsubseteq} F_{2A}] = \inf_{e_M \tilde{\in} X_A \backslash F_{2A}} SN_{e_M}(X_A \backslash (F_{1A}))$. So $[F_{2A} \dot{\equiv} semi - Cl(F_{1A})] = \max(0, \inf_{e_M \tilde{\in} X_A \backslash F_{2A}} SN_{e_M}(X_A \backslash (F_{1A})) - \sup_{e_M \tilde{\in} F_{2A} \backslash F_{1A}} SN_{e_M}(X_A \backslash (F_{1A})))$. If $[F_{2A} \dot{\equiv} semi - Cl(F_{1A})] > t$, then $\inf_{e_M \tilde{\in} X_A \backslash F_{2A}} SN_{e_M}(X_A \backslash (F_{1A})) > t + \sup_{e_M \tilde{\in} F_{2A} \backslash F_{1A}} SN_{e_M}(X_A \backslash (F_{1A}))$. For any $e_M \tilde{\in} X_A \backslash F_{2A}$, we have $\sup_{e_M \tilde{\in} F_{3A} \tilde{\sqsubseteq} F_{2A} \backslash F_{1A}} S\tilde{\tau}(F_{3A}) > t + \sup_{e_M \tilde{\in} F_{2A} \backslash F_{1A}} SN_{e_M}(X_A \backslash (F_{1A}))$, i.e., there exists $F_{3A}(e_M)$ such that
$e_M \tilde{\in} F_{3A} \tilde{\sqsubseteq} X_A \backslash F_{1A}$ and $S\tilde{\tau}(F_{3A}) > t + \sup_{e_M \tilde{\in} F_{2A} \backslash F_{1A}} SN_{e_M}(X_A \backslash (F_{1A}))$. Now we need to prove that $F_{3A}(e_M) \tilde{\sqsubseteq} X_A \backslash (F_{2A})$. If not, then there exists $e_{Mi} \tilde{\in} F_{1A} \backslash F_{2A}$ with $e_{Mi} \tilde{\in} F_{3A}(e_M)$. Hence we obtain $\sup_{e_M \tilde{\in} F_{2A} \backslash F_{1A}} SN_{e_M}(X_A \backslash (F_{1A})) \geq SN_{e_{Mi}}(X_A \backslash (F_{1A})) \geq S\tilde{\tau}(F_{3A}(e_M)) > t + \sup_{e_M \tilde{\in} F_{2A} \backslash F_{1A}} SN_{e_M}(X_A \backslash (F_{1A}))$, a contradiction. Therefore $S\tilde{\mathcal{F}}(F_{2A}) = S\tilde{\tau}(X_A \backslash (F_{2A})) = \inf_{e_M \tilde{\in} X_A \backslash F_{2A}} SN_{e_M}(X_A \backslash (F_{2A})) \geq \inf_{e_M \tilde{\in} X_A \backslash F_{2A}} S\tilde{\tau}(F_{3A}) > t + \sup_{e_M \tilde{\in} F_{2A} \backslash F_{1A}} SN_{e_M}(X_A \backslash (F_{1A})) > t$. Since t is arbitrary, it holds that $[F_{2A} \dot{\equiv} semi - Cl(F_{1A})] \leq [F_{2A} \tilde{\in} S\tilde{\mathcal{F}}]$.

(2)From part (2) of Theorem 3.4, we have $semi - Cl(F_{1A})(e_M) = \inf_{e_M \notin F_{2A} \tilde{\sqsupseteq} F_{1A}} (1 - S\tilde{\mathcal{F}}(F_{2A})) \leq \inf_{e_M \notin F_{2A} \tilde{\sqsupseteq} F_{1A}} (1 - \tilde{\mathcal{F}}(F_{2A})) = Cl(F_{1A})$.

(3)Let $F_{1A} \tilde{\sqsubseteq} F_{2A}$, then $X_A \backslash F_{2A} \tilde{\sqsubseteq} X_A \backslash F_{1A}$, we have $semi - Cl(F_{1A})(e_M) = 1 - SN_{e_M}(X_A \backslash F_{1A}) \leq 1 - SN_{e_M}(X_A \backslash F_{2A}) = semi - Cl(F_{2A})(e_M)$.

Definition 3.26. Let $(X, \tilde{\tau}, A)$ be a fuzzifying soft topological space. A semi-interior of a soft set F_A is denoted by $semi - Int(F_A)$ and defined as follows;

$$semi - Int(F_A)(e_M) = SN_{e_M}(F_A).$$

Theorem 3.27. Let $(X, \tilde{\tau}, A)$ be a fuzzifying soft topological space,
$F_{1A}, F_{2A} \tilde{\in} SS(X_A)$, and $e_F \tilde{\in} X_A$. Then
(1) $\models semi - Int(X_A) \equiv X_A$;
(2) $\models semi - Int(F_{1A}) \tilde{\sqsubseteq} F_{1A}$;
(3) $\models Int(F_{1A}) \tilde{\sqsubseteq} semi - Int(F_{1A})$;
(4) $\models F_{2A} \tilde{\in} S\tilde{\tau} \wedge F_{2A} \tilde{\sqsubseteq} F_{1A} \longrightarrow F_{2A} \tilde{\sqsubseteq} semi - Int(F_{1A})$;
(5) $\models F_{1A} \equiv semi - Int(F_{1A}) \leftrightarrow F_{1A} \tilde{\in} S\tilde{\tau}$;
(6) $\models F_{1A} \tilde{\sqsubseteq} F_{2A} \longrightarrow semi - Int(F_{1A}) \tilde{\sqsubseteq} semi - Int(F_{2A})$;
(7) $\models semi - Int(F_{1A}) \equiv (X_A \backslash semi - Cl(X_A \backslash F_{1A}))$;
(8) $\models semi - Int(F_{1A}) \equiv F_{1A} \tilde{\sqcap} (X_A \backslash \text{semi} - d(X_A \backslash F_{1A}))$;
(9) $\models F_{2A} \dot{\equiv} semi - Int(F_{1A}) \longrightarrow F_{2A} \tilde{\in} S\tilde{\tau}$;
(10) $\models e_M \tilde{\in} semi - Int(F_{1A}) \leftrightarrow e_M \tilde{\in} F_{1A} \wedge e_M \tilde{\in} (X_A \backslash semi - d(X_A \backslash F_{1A}))$;

Proof.
(1) $semi-Int(X_A)(e_M)=SN_{e_M}(X_A)=1$. For all $e_M \tilde{\in} X_A$ Therefore $semi-Int(X_A)\equiv X_A$.
(2) Let $F_{1A}\tilde{\in} SS(X_A)$, and $e_M\tilde{\in} X_A$. If $e_M\notin F_{1A}$, then $semi-Int(F_{1A})(e_M)=SN_{e_M}(F_{1A})=0$. Therefore
$semi-Int(F_{1A})\tilde{\sqsubseteq} F_{1A}$.
(3) From (2) of Theorem 3.12, we have $Int(F_{1A})(e_M)=N_{e_M}(F_{1A})\le SN_{e_M}(F_{1A})=semi-Int(F_{1A})(e_M)$.
(4) If $F_{2A}\tilde{\not\sqsubseteq} F_{1A}$, then $[F_{2A}\tilde{\in} S\tilde{\tau}\wedge F_{2A}\tilde{\sqsubseteq} F_{1A}]=0$. If $F_{2A}\tilde{\sqsubseteq} F_{1A}$, then $[F_{2A}\tilde{\sqsubseteq} semi-Int(F_{1A})]=\inf_{e_M\tilde{\in}F_{2A}} semi-Int(F_{1A})(e_M)=\inf_{e_M\tilde{\in}F_{2A}} SN_{e_M}(F_{1A})\ge \inf_{e_M\tilde{\in}F_{2A}} SN_{e_M}(F_{2A})=S\tilde{\tau}(F_{2A})=[F_{2A}\tilde{\in} S\tilde{\tau}\wedge F_{2A}\tilde{\sqsubseteq} F_{1A}]$.
(5) We have $[F_{1A}\equiv semi-Int(F_{1A})]=\min(\inf_{e_M\tilde{\in}F_{1A}} semi-Int(F_{1A})(e_M), \inf_{e_M\tilde{\in}(X_A\setminus(F_{1A}))}(1-semi-Int(F_{1A})(e_M)))=\inf_{e_M\tilde{\in}F_{1A}}(semi-Int(F_{1A})(e_M))=\inf_{e_M\tilde{\in}F_{1A}} SN_{e_M}(F_{1A})=S\tilde{\tau}(F_{1A})=[F_{1A}\tilde{\in} S\tilde{\tau}]$
(6) From Definition 3.26 and (2) of Theorem 3.14, the proof is straightforward.
(7) From (1) of Theorem 3.22, we have $(X_A\setminus semi-Cl(X_A\setminus F_{1A}))(e_M)=1-(1-SN_{e_M}(F_{1A}))=SN_{e_M}(F_{1A})=semi-Int(F_{1A}))(e_M)$.
(8) From Lemma 3.17, we have
$[F_{1A}\tilde{\sqcap}(X_A\setminus \text{semi}-d(X_A\setminus F_{1A}))]=\min(F_{1A}(e_M),SN_{e_M}(F_{1A}\tilde{\sqcup}\{e_M\}))$
If $e_M\notin F_{1A}$, then $[F_{1A}\tilde{\sqcap}(X_A\setminus \text{semi}-d(X_A\setminus F_{1A}))]=0=SN_{e_M}(F_{1A})=semi-Int(F_{1A}))(e_M)$. If $e_M\tilde{\in} F_{1A}$, then $[F_{1A}\tilde{\sqcap}(X_A\setminus \text{semi}-d(X_A\setminus F_{1A}))]=SN_{e_M}(F_{1A})=semi-Int(F_{1A}))(e_M)$.
(9) From(7) of Theorem 3.27 and (1) of Theorem 3.25, we have
$[F_{2A}\doteqdot semi-Int(F_{1A})\to F_{2A}\tilde{\in} S\tilde{\tau}]=$ $[X_A\setminus F_{2A}\doteqdot semi-Cl(\{X_A\setminus F_{1A})]\le[X_A\setminus F_{2A}\tilde{\in} S\tilde{\mathcal{F}}]=[F_{2A}\tilde{\in} S\tilde{\tau}]$.
(10) If $e_M\notin F_{1A}$, then $[e_M\tilde{\in} semi-Int(F_{1A})]=0=[e_M\tilde{\in} F_{1A}\wedge e_M\tilde{\in}(X_A\setminus semi-d(X_A\setminus F_{1A}))]$, if $e_M\tilde{\in} F_{1A}$, then $[e_M\tilde{\in} semi-d(F_{1A})]=1-SN_{e_M}(F_{1A}\tilde{\sqcup} e_F)=1-SN_{e_M}(F_{1A})=1-semi-Int(F_{1A})(e_M)$, so that
$[e_M\tilde{\in} F_{1A}\wedge e_F\tilde{\in}(X_A\setminus semi-d(X_A\setminus F_{1A}))]=[e_M\tilde{\in} semi-Int(F_{1A})]$.

Definition 3.28. Let $(X,\tilde{\tau},A)$ be a fuzzifying soft topological space. The exterior of a soft set F_A is denoted by $ext(F_A)$ and defined as follows;
$e_F\tilde{\in} ext(F_A):=e_M\tilde{\in} Int(X_A\setminus F_A)$, i.e., $ext(F_A)(e_M)=Int(X_A\setminus F_A)(e_M)$.

Definition 3.29. Let $(X,\tilde{\tau},A)$ be a fuzzifying soft topological space. The semi-exterior of a soft set F_A is denoted by $semi-ext(F_A)$ and defined as follows;
$e_M\tilde{\in} semi-ext(F_A):=e_M\tilde{\in} semi-Int(X_A\setminus F_A)$, i.e., $semi-ext(F_A)(e_M)=semi-Int(X_A\setminus F_A)(e_M)$.

Theorem 3.30. $(X,\tilde{\tau},A)$ be a fuzzifying soft topological space,
$F_{1A},F_{2A}\tilde{\in} SS(X_A)$, and $e_M\tilde{\in} X_A$. Then
(1) $\models semi-ext(\Phi_A)\equiv X_A$;
(2) $\models semi-ext(F_{1A})\tilde{\sqsubseteq} X_A\setminus F_{1A}$;
(3) $\models ext(F_{1A})\tilde{\sqsubseteq} semi-ext(F_{1A})$;
(4) $\models F_{1A}\tilde{\in} S\tilde{\mathcal{F}}\leftrightarrow semi-ext(F_{1A})\equiv X_A\setminus F_{1A}$;

(5) $\models F_{2A} \widetilde{\in} S\widetilde{\mathcal{F}} \wedge F_{1A} \widetilde{\sqsubseteq} F_{2A} \longrightarrow X_A \backslash F_{2A} \widetilde{\sqsubseteq} semi-ext(F_{1A})$;
(6) $\models F_{2A} \widetilde{\sqsubseteq} F_{1A} \longrightarrow semi-ext(F_{2A}) \widetilde{\sqsubseteq} semi-ext(F_{1A})$;
(7) $\models semi-ext(F_{1A}) \equiv (X_A \backslash F_{1A}) \widetilde{\sqcap} (X_A \backslash semi-d(F_{1A}))$;
(8) $\models semi-ext(F_{1A}) \equiv (X_A \backslash semi-Cl(F_{1A}))$;
(9) $\models e_M \widetilde{\in} semi-ext(F_{1A}) \leftrightarrow \exists F_{2A}(e_M \widetilde{\in} F_{2A} \widetilde{\in} S\tilde{\tau} \wedge F_{2A} \widetilde{\sqcap} F_{1A} = \Phi_A)$.

Proof. From Theorem 3.27, we can prove (1),(2),(3),(4),(5),(6),(7) and (8).
(9)$[\exists F_{2A}(e_M \widetilde{\in} F_{2A} \widetilde{\in} S\tilde{\tau} \wedge F_{2A} \widetilde{\sqcap} F_{1A} = \Phi_A)] = \sup_{e_M \widetilde{\in} F_{2A} \widetilde{\sqsubseteq} X_A \backslash (F_{1A})} S\tilde{\tau}(F_{2A}) = SN_{e_M}(X_A \backslash (F_{1A})) = semi-Int(X_A \backslash (F_{1A}))(e_M)$.

Definition 3.31. Let $(X, \tilde{\tau}, A)$ be a fuzzifying soft topological space and $F_A \widetilde{\sqsubseteq} X_A$. The semi-boundary of a soft set F_A is denoted by $semi-b(F_A)$ and defined as follows;

$$e_M \widetilde{\in} semi-b(F_A) := (e_M \notin semi-Int(F_A) \wedge e_M \notin semi-Int(X_A \backslash F_A)),$$

i.e., $semi-b(F_A)(e_M) = \min(1-(semi-Int(F_A)), 1-(semi-Int(X_A \backslash F_A)))(e_M)$.

Example 3.32. Let us consider the fuzzifying soft toplogical space $(X, \tilde{\tau}, A)$ in Example 3.11. For the soft set $F_A = \{(e_1, \{a\}), (e_2, \{a, b\})\}$ we have $semi-Int(F_A)(e_1/a) = \sup_{e_1/a \widetilde{\in} H_A \widetilde{\sqsubseteq} F_A} S\tilde{\tau}(H_A) = \frac{3}{4}$. Similarly $semi-Int(F_A)(e_1/b) = 0, semi-Int(F_A)(e_2/a) = \frac{3}{4}$ and semi-Int$(F_A)(e_2/b) = \frac{3}{4}$.Hence
$semi-Int(F_A) = \{(e_1, \{a/\frac{3}{4}, b/0\}), (e_2, \{a/\frac{3}{4}, b/\frac{3}{4}\})\}$, $semi-ext(F_A)(e_1/a) = semi-Int(X_A \backslash F_A)(e_1/a)$. Then $semi-ext(F_A)(e_1/a) = \sup_{e_1/a \widetilde{\in} H_A \widetilde{\sqsubseteq} (X_A \backslash F_A)} S\tilde{\tau}(H_A) = 0$. Similarly $semi-ext(F_A)(e_1/b) = 0$
$, semi-ext(F_A)(e_2/a) = 0, semi-ext(F_A)(e_2/b) = 0$, hence $semi-b(F_A)(e_1/a) = \min((1-semi-Int(F_A)), (1-semi-Int(X_A \backslash F_A)))(e_1/a) = \frac{1}{4}$. Similarly $semi-b(F_A)(e_1/b) = 1$, $semi-b(F_A)(e_2/a) = \frac{1}{4}$, $semi-b(F_A)(e_2/b) = \frac{1}{4}$. Therfore $semi-b(F_A) = \{(e_1, \{a/\frac{1}{4}, b/1\}), (e_2, \{a/\frac{1}{4}, b/\frac{1}{4}\})\}$ which is a fuzzy soft set.

Lemma 3.33. $\models e_M \widetilde{\in} semi-b(F_A) \leftrightarrow \forall F_{2A}(F_{2A} \widetilde{\in} SN_{e_M} \rightarrow (F_{2A} \widetilde{\sqcap} F_A \neq \Phi_A) \wedge (F_{2A} \widetilde{\sqcap} (X_A \backslash F_A) \neq \Phi_A))$, for any $F_A \widetilde{\in} SS(X_A)$ and $e_M \widetilde{\in} X_A$.

Proof. $[\forall F_{2A}(F_{2A} \widetilde{\in} SN_{e_M} \rightarrow (F_{2A} \widetilde{\sqcap} F_A \neq \Phi_A) \wedge (F_{2A} \widetilde{\sqcap} (X_A \backslash F_A) \neq \Phi_A))]$

$$= min(\inf_{F_{2A} \widetilde{\sqsubseteq} F_A} (1 - SN_{e_M}(F_{2A})), \inf_{F_{2A} \widetilde{\sqsubseteq} X_A \backslash F_A} (1 - SN_{e_M}(F_{2A})))$$
$$= min(1 - SN_{e_M}(F_A), 1 - SN_{e_M}(X_A \backslash F_A))$$
$$= min(1 - semi-Int(F_A)(e_M), 1 - semi-Int(X_A \backslash F_A)(e_M))$$
$$= [e_M \widetilde{\in} semi-b(F_A)].$$

Theorem 3.34. $(X, \tilde{\tau}, A)$ be a fuzzifying soft topological space, $F_A \widetilde{\in} SS(X_A)$, and $e_M \widetilde{\in} X_A$. Then.
(1) $\models semi-b(F_A) \equiv semi-Cl(F_A) \widetilde{\sqcap} semi-Cl(X_A \backslash F_A)$;
(2) $\models semi-b(F_A) \equiv semi-b(X_A \backslash F_A)$;
(3) $\models X_A \backslash semi-b(F_A) \equiv semi-Int(F_A) \widetilde{\sqcup} semi-Int(X_A \backslash F_A)$;
(4) $\models semi-Cl(F_A) \equiv F_A \widetilde{\sqcup} semi-b(F_A)$;
(5) $\models semi-b(F_A) \widetilde{\sqsubseteq} F_A \leftrightarrow F_A \widetilde{\in} S\widetilde{\mathcal{F}}$;

(6) $\models semi-Int(F_A) \equiv F_A \tilde{\sqcap} (X_A \backslash semi-b(F_A))$;
(7) $\models (semi-b(F_A) \tilde{\sqcap} F_A \equiv \Phi_A) \leftrightarrow F_A \tilde{\in} S\tilde{\tau}$;
(8) $\models semi-b(F_A) \tilde{\sqsubseteq} b(F_A)$;
(9) $\models X_A \backslash semi-b(F_A); \equiv semi-Int(F_A) \tilde{\sqcup} semi-ext(F_A)$.

Proof.

(1) From (7) of Theorem 3.27, we obtain $(semi-Cl(F_A) \tilde{\sqcap} semi-Cl(X_A \backslash F_A))(e_M) = \min(semi-Cl(F_A)(e_M), semi-Cl(X_A \backslash F_A)(e_M)) = \min(1-(semi-Int(X_A \backslash F_A))(e_M), 1-(semi-Int(F_A)(e_M))) = semi-b(F_A)(e_M)$.

(2) Straightforward.

(3) From (1) and (7) of Theorem 3.27, we obtain
$X_A \backslash semi-b(F_A) \equiv X_A \backslash (semi-Cl(F_A) \tilde{\sqcap} semi-Cl(X_A \backslash F_A)) \equiv X_A \backslash semi-Cl(F_A) \tilde{\sqcup} (X_A \backslash semi-Cl(X_A \backslash F_A)) \equiv semi-Int(X_A \backslash F_A) \tilde{\sqcup} semi-Int(F_A)$

(4) If $e_M \tilde{\in} F_A$, then $semi-Cl(F_A)(e_M) = 1 = (F_A \tilde{\sqcup} semi-b(F_A))(e_M)$.

If $e_M \notin F_A$, then $(F_A \tilde{\sqcup} semi-b(F_A))(e_M) = semi-b(F_A)(e_M) = \min(1-(semi-Int(F_A))(e_M), 1-(semi-Int(X_A \backslash F_A))(e_M)) = 1-semi-Int(X_A \backslash F_A)(e_M) = semi-Cl(F_A)(e_M)$.

(5) From (3) of Theorem 3.18,(2) of Theorem 3.24, Lemma 3.17 and (4) above, we obtain

$$\begin{aligned} F_A \tilde{\in} S\tilde{\mathcal{F}} &\leftrightarrow semi-d(F_A) \tilde{\sqsubseteq} F_A \\ &\leftrightarrow F_A \tilde{\sqcup} semi-d(F_A) \tilde{\sqsubseteq} F_A \\ &\leftrightarrow semi-Cl(F_A) \tilde{\sqsubseteq} F_A \\ &\leftrightarrow F_A \tilde{\sqcup} semi-b(F_A) \tilde{\sqsubseteq} F_A \\ &\leftrightarrow semi-b(F_A) \tilde{\sqsubseteq} F_A. \end{aligned}$$

(6) from (7) of Theorem 3.27 and (4) above, we obtain

$$\begin{aligned} semi-Int(F_A) &\equiv X_A \backslash semi-Cl(X_A \backslash F_A) \\ &\equiv X_A \backslash (X_A \backslash F_A \tilde{\sqcup} semi-b(X_A \backslash F_A)) \\ &\equiv F_A \tilde{\sqcap} (X_A \backslash semi-b(X_A \backslash F_A)) \\ &\equiv F_A \tilde{\sqcap} (X_A \backslash semi-b(F_A)). \end{aligned}$$

(7) from (5) of Theorem 3.27 and (6) above , we obtain

$$\begin{aligned} semi-b(F_A) \tilde{\sqcap} F_A \equiv \Phi_A &\leftrightarrow (X_A \backslash semi-b(F_A)) \tilde{\sqcup} (X_A \backslash F_A) \equiv X_A \\ &\leftrightarrow F_A \tilde{\sqsubseteq} X_A \backslash semi-b(F_A) \\ &\leftrightarrow F_A \tilde{\sqcap} (X_A \backslash semi-b(F_A)) \equiv F_A \\ &\leftrightarrow semi-Int(F_A) \equiv F_A \leftrightarrow F_A \tilde{\in} S\tilde{\tau}. \end{aligned}$$

(8) from (3) of Theorem 3.27, we have

$$\begin{aligned} &semi-b(F_A)(e_M) \\ &= \min(1-(semi-Int(F_A))(e_M), 1-(semi-Int(X_A \backslash F_A))(e_M)) \\ &\leq \min(1-Int(F_A)(e_{MF}), 1-Int(X_A \backslash F_A)(e_M)) = b(F_A)(e_M). \end{aligned}$$

(9) From (3), we have

$X_A \backslash semi-b(F_A) \equiv semi-Int(F_A) \tilde{\sqcup} semi-Int(X_A \backslash (F_A)) \equiv semi-Int(F_A) \tilde{\sqcup} semi-ext(F_A)$.

4. Soft Semi-Continuous Functions in Fuzzifying Soft Topological

In this section, we introduce and study the concept of soft Semi-continuous functions in fuzzifying soft topological space with investigating some of its properties

Definition 4.1. Let $(X, \tilde{\tau}, A)$ and $(Y, \tilde{\sigma}, B)$ be two fuzzifying soft topological spaces, $u: X \to Y$ and

$p: A \to B$ be mappings. The fuzzifying soft semi-continuity of a mapping $f_{pu}: SS(X_A) \to SS(Y_B)$ denoted by $SC \tilde{\in} \Im(S(Y,B)^{S(X,A)})$, is defined as follows
$SC(f_{pu}) := (\forall F_B)((F_B \tilde{\in} \tilde{\sigma}) \to (f_{pu}^{-1}(F_B) \tilde{\in} S\tilde{\tau}))$. Intuitively, the degree to which f_{pu} is a fuzzifying soft semi-continuous is $[SC(f_{pu})] = \inf_{F_B \tilde{\in} SS(Y_B)} \min(1, 1 - \tilde{\sigma}(F_B) + S\tilde{\tau}(f_{pu}^{-1}(F_B)))$.

Theorem 4.2. *Let $(X, \tilde{\tau}, A)$ and $(Y, \tilde{\sigma}, B)$ be two fuzzifying soft topological spaces, then* $\models C(f_{pu}) \to SC(f_{pu})$, *for each* $(f_{pu}) \tilde{\in} \Im(S(Y,B)^{S(X,A)})$,
Proof. The proof is obtained from (1) of Theorem 3.4
The converse of Theorem 4.2 generally need not be true as shown by the following example:

Example 4.3. Let $(X, \tilde{\tau}, A)$ be a fuzzifying soft topological space defined in Example 3.5, consider the identity function (f_{pu}) from $(X, \tilde{\tau}, A)$ onto $(X, \tilde{\sigma}, A)$ where $\tilde{\sigma}$ is fuzzifying soft topology on X defined as follows;

$$\tilde{\sigma}(F_A) = \begin{cases} 1 & if \quad F_A \tilde{\in} \{\Phi_A, X_A, \{(e_1,\{x_2\}), (e_2,\{x_2\}\} \\ 0 & \text{otherwise.} \end{cases}$$

Note that if $G_A = Y_A$ or Φ_A or $f_{pu}(G_A) = 0$. Then $\min(1, 1 - \tilde{\sigma}(G_A) + S\tilde{\tau}f^{-1}{}_{pu}(G_A)) = 1$. Thus those cases are rejected, since we are looking for the minimum value, will the other cases which are only a soft set $\{(e_1,\{x_2\}), (e_2,\{x_2\}$, we discuss. $C(f_{pu}) = \min(1, 1 - \tilde{\sigma}(\{(e_1,\{x_2\}), (e_2,\{x_2\}\}) + \tilde{\tau}f^{-1}{}_{pu}(\{(e_1,\{x_2\}), (e_2,\{x_2\}\}))$
$= \min(1, 1 - 1 + 0) = 0$ and
$SC(f_{pu}) = \min(1, 1 - \tilde{\sigma}(\{(e_1,\{x_2\}), (e_2,\{x_2\}\}) + S\tilde{\tau}f^{-1}{}_{pu}(\{(e_1,\{x_2\}), (e_2,\{x_2\}\}))$
$= \min(1, 1 - 1 + \frac{3}{4}) = \frac{3}{4}$ Then $SC(f_{pu}) = \frac{3}{4}$, $C(f_{pu}) = 0$. Hence the statement $SC(f_{pu}) \to C(f_{pu})$ is not true.

Theorem 4.4 *Let* $(X, \tilde{\tau}, A)$, $(Y, \tilde{\sigma}, B)$ *and* $(Z, \tilde{V}, C)$ *be three fuzzifying soft topological spaces,* $(f_{pu}) \tilde{\in} \Im(S(Y,B)^{S(X,A)})$ and $(g_{bd}) \tilde{\in} \Im(S(Z,C)^{S(Y,B)})$. Then
(1) $\models SC(f_{pu}) \to (C(g_{bd}) \to SC(g_{bd} \circ f_{pu}))$.
(2) $\models C(g_{bd}) \to (SC(f_{pu}) \to SC(g_{bd} \circ f_{pu}))$.
Proof. (1) We need to prove that $[SC(f_{pu})] \le [C(g_{bd}) \to SC(g_{bd} \circ f_{pu})]$. If $[C(g_{bd})] \le [SC(g_{bd} \circ f_{pu})]$, the result holds, if $[C(g_{bd})] > [SC(g_{bd} \circ f_{pu})]$, then $[C(g_{bd})] - [SC(g_{bd} \circ f_{pu})] = \inf_{F_C \tilde{\in} SS(Z_C)} \min(1, (1 - \tilde{V}(F_C) + \tilde{\sigma}(g^{-1}{}_{bd}(F_C))) - \inf_{F_C \tilde{\in} SS(Z_C)} \min(1, 1 - \tilde{V}(F_C) + S\tilde{\tau}(g_{bd} \circ f_{pu})^{-1}(F_C))) \le \sup_{F_C \tilde{\in} SS(Z_C)} (\tilde{\sigma}(g^{-1}{}_{bd}(F_C)) - S\tilde{\tau}((g_{bd} \circ f_{pu})^{-1}(F_C))) \le \sup_{F_B \tilde{\in} SS(Y_B)} (\tilde{\sigma}(F_B) - S\tilde{\tau}(f^{-1}{}_{pu}(F_B)))$. Therefore, $[C(g_{bd}) \to SC(g_{bd} \circ f_{pu})] = \min(1, 1 - C(g_{bd}) + SC(g_{bd} \circ f_{pu})) \ge \inf_{F_B \tilde{\in} SS(Y_B)} \min(1, 1 - \tilde{\sigma}(F_B) + S\tilde{\tau}(f^{-1}{}_{pu}(F_B))) = SC(f_{pu})$.
*(2)*We need to prove that $[C(g_{bd})] \le [SC(f_{pu}) \to SC(g_{bd} \circ f_{pu})]$. If $[SC(f_{pu})] \le [SC(g_{bd} \circ f_{pu})]$, the result holds, if $[SC(f_{pu})] > [SC(g_{bd} \circ f_{pu})]$, then $[SC(f_{pu})] - [SC(g_{bd} \circ f_{pu})] = \inf_{F_B \tilde{\in} SS(Y_B)} \min(1, 1 - \tilde{\sigma}(F_B) + S\tilde{\tau}(f^{-1}{}_{pu}(F_B))) - \inf_{F_C \tilde{\in} SS(Z_C)} \min(1, 1 - \tilde{V}(F_C) + S\tilde{\tau}(g_{bd} \circ f_{pu})^{-1}(F_C))) \le$
$\sup_{F_C \tilde{\in} SS(Z_C)} (-\tilde{\sigma}(g^{-1}{}_{bd}(F_C)) + S\tilde{\tau}(f^{-1}{}_{pu}(g^{-1}{}_{bd}(F_C))) + \tilde{V}(F_C) - S\tilde{\tau}(f^{-1}{}_{pu}\left(g^{-1}{}_{bd}(F_C)\right)))$

$\leq \sup_{F_C \tilde{\in} SS(Z_C)} (\tilde{\nabla}(F_C) - \tilde{\sigma}(g^{-1}{}_{bd}(F_C)))$. Therefore, $[SC(f_{pu}) \rightarrow SC(g_{bd} \circ f_{pu})] = \min(1, 1 - SC(f_{pu}) + SC(g_{bd} \circ f_{pu})) \geq \inf_{F_C \tilde{\in} SS(Z_C)} \min(1, 1 - \tilde{\nabla}(F_C) + \tilde{\sigma}(g^{-1}{}_{bd}(F_C))) = C(g_{bd})$.

Theorem 4.5. Let $(X, \tilde{\tau}, A)$, $(Y, \tilde{\sigma}, B)$ and $(Z, \tilde{V}, C)$ be three fuzzifying soft topological spaces, $(f_{pu}) \tilde{\in} \Im$
$(S(Y, B)^{S(X,A)})$ and $(g_{bd}) \tilde{\in} \Im(S(Z, C)^{S(Y,B)})$. Then the following statements are equivalent.
(1) $\vDash SC(f_{pu}) \rightarrow (C(g_{bd}) \rightarrow SC(g_{bd} \circ f_{pu}))$.
(2) $\vDash C(g_{bd}) \rightarrow (SC(f_{pu}) \rightarrow SC(g_{bd} \circ f_{pu}))$.
Proof. Since the conjunction $\wedge$ is commutative, then

$$\begin{aligned}
&[SC(f_{pu}) \rightarrow (C(g_{bd}) \rightarrow SC(g_{bd} \circ f_{pu}))] \\
&= [SC(f_{pu}) \rightarrow \neg(C(g_{bd}) \wedge \neg(SC(g_{bd} \circ f_{pu})))] \\
&= [\neg(SC(f_{pu}) \wedge \neg\neg(C(g_{bd}) \wedge \neg(SC(g_{bd} \circ f_{pu}))))] \\
&= [\neg(SC(f_{pu}) \wedge C(g_{bd}) \wedge \neg(SC(g_{bd} \circ f_{pu})))] \\
&= [\neg(C(g_{bd}) \wedge SC(f_{pu}) \wedge \neg(SC(g_{bd} \circ f_{pu})))] \\
&= [\neg(C(g_{bd}) \wedge \neg\neg(SC(f_{pu}) \wedge \neg SC(g_{bd} \circ f_{pu})))] \\
&= [C(g_{bd}) \rightarrow \neg(SC(f_{pu}) \wedge \neg SC(g_{bd} \circ f_{pu}))] \\
&= [C(g_{bd}) \rightarrow (SC(f_{pu}) \rightarrow SC(g_{bd} \circ f_{pu}))].
\end{aligned}$$

The proofs of (2 → 1) is approximately similar to the proof (1 → 2).

Definition 4.6. Let $(X, \tilde{\tau}, A)$ and $(Y, \tilde{\sigma}, B)$ be two fuzzifying soft topological spaces, $u: X \rightarrow Y$ and $p: A \rightarrow B$ be mappings. The fuzzifying soft functions $SC\alpha_j \tilde{\in} \Im(S(Y, B)^{S(X,A)})$, *where* $j = 1,2, \ldots, 5$ are defined as follows;
(1) $SC\alpha_1(f_{pu}) := \forall F_B(F_B \tilde{\in} \tilde{\mathrm{F}}) \rightarrow (f_{pu}^{-1}(F_B) \tilde{\in} S\tilde{\mathrm{F}})$;
where $\tilde{\mathrm{F}}$ is the family of soft closed subsets of Y; and $S\tilde{\mathrm{F}}$ is a Family of soft semi-closed subsets of X.
(2) $SC\alpha_2(f_{pu}) := \forall e_F \forall F_B(F_B \tilde{\in} N_{f_{pu}(e_M)} \rightarrow f_{pu}^{-1}(F_B) \tilde{\in} SN_{e_M})$;
where N is the family of soft neighborhood systems of Y; and SN is a soft semi-neighborhood systems of X.
(3) $SC\alpha_3(f_{pu}) := \forall e_M \forall F_B(F_B \tilde{\in} N_{f_{pu}(e_M)} \rightarrow \exists F_A(f_{pu}(F_A) \tilde{\sqsubseteq} F_B \rightarrow F_A \tilde{\in} SN_{e_M}))$;
(4) $SC\alpha_4(f_{pu}) := \forall F_A(f_{pu}(semi - Cl_X(F_A)) \tilde{\sqsubseteq} Cl_Y(f_{pu}(F_A)))$;
(5) $SC\alpha_5(f_{pu}) := \forall F_B(semi - Cl_X(f_{pu}^{-1}(F_B)) \tilde{\sqsubseteq} f_{pu}^{-1}(Cl_Y(F_B)))$;

Theorem 4.7. Let $(X, \tilde{\tau}, A)$ and $(Y, \tilde{\sigma}, B)$ be two fuzzifying soft topological spaces, $u: X \rightarrow Y$ and $p: A \rightarrow B$ be mappings. Then
$\vDash f_{pu} \tilde{\in} SC(f_{pu}) \leftrightarrow f_{pu} \tilde{\in} SC\alpha_1(f_{pu})$;

Proof.

$[f_{pu} \tilde{\in} SC\alpha_1(f_{pu})] = \inf_{F_B \tilde{\in} SS(Y_B)} \min(1, 1 - \tilde{\mathrm{F}}(F_B) + S\tilde{\mathrm{F}}(f_{pu}^{-1}(F_B)))$
$= \inf_{F_B \tilde{\in} SS(Y_B)} \min(1, 1 - \tilde{\sigma}(Y_B \backslash F_B) + S\tilde{\tau}(X_A \backslash (f_{pu}^{-1}(F_B))))$
$= \inf_{F_B \tilde{\in} SS(Y_B)} \min(1, 1 - \tilde{\sigma}(Y_B \backslash F_B) + S\tilde{\tau}(f_{pu}^{-1}(Y_B \backslash (F_B))))$
$= \inf_{F_{1B} \tilde{\in} SS(Y_B)} \min(1, 1 - \tilde{\sigma}(F_{1B}) + S\tilde{\tau}(f_{pu}^{-1}(F_{1B})))$

$=[f_{pu} \tilde{\in} SC(f_{pu})]$, Where $Y_B \setminus F_B = F_{1B}$.

Theorem 4.8. Let $(X, \tilde{\tau}, A)$ and $(Y, \tilde{\sigma}, B)$ be two fuzzifying soft topological spaces, $u: X \to Y$ and $p: A \to B$ be mappings. Then

$\vDash f_{pu} \tilde{\in} SC(f_{pu}) \leftrightarrow f_{pu} \tilde{\in} SC\alpha_2(f_{pu})$;

Proof.

Firstly, we prove that $SC\alpha_2(f_{pu}) \geq SC(f_{pu})$. If $N_{f_{pu}(e_M)}(F_B) \leq SN_{e_M}(f_{pu}^{-1}(F_B))$, then the result holds. Suppose $N_{f_{pu}(e_M)}(F_B) > SN_{e_M}(f_{pu}^{-1}(F_B))$. It is clear that if, $f_{pu}(e_M) \tilde{\in} F_{1B} \tilde{\sqsubseteq} F_B, e_M \tilde{\in} f_{pu}^{-1}(F_{1B}) \tilde{\sqsubseteq} f_{pu}^{-1}(F_B)$. Then $N_{f_{pu}(e_M)}(F_B) - SN_{e_M}(f_{pu}^{-1}(F_B)) = \sup_{f_{pu}(e_M) \tilde{\in} F_{1B} \tilde{\sqsubseteq} F_B} \tilde{\sigma}(F_{1B}) - \sup_{(e_M) \tilde{\in} F_A \tilde{\sqsubseteq} f_{pu}^{-1} F_B} (S\tilde{\tau}(F_A)) \leq \sup_{f_{pu}(e_M) \tilde{\in} F_{1B} \tilde{\sqsubseteq} F_B} \tilde{\sigma}(F_{1B}) - \sup_{f_{pu}(e_M) \tilde{\in} F_{1B} \tilde{\sqsubseteq} F_B} S\tilde{\tau}(f_{pu}^{-1}(F_{1B})) \leq \sup_{f_{pu}(e_M) \tilde{\in} F_{1B} \tilde{\sqsubseteq} F_B} (\tilde{\sigma}(F_{1B}) - S\tilde{\tau}(f_{pu}^{-1}(F_{1B})))$. So $1 - N_{f_{pu}(e_M)}(F_B) + SN_{e_M}(f_{pu}^{-1}(F_B)) \geq \inf_{f_{pu}(e_M) \tilde{\in} F_{1B} \tilde{\sqsubseteq} F_B} (1 - \tilde{\sigma}(F_{1B}) + S\tilde{\tau}(f_{pu}^{-1}(F_{1B})))$ and thus $\min(1 - N_{f_{pu}(e_M)}(F_B) + SN_{e_M}(f_{pu}^{-1}$
$(F_B))) \geq \inf_{f_{pu}(e_M) \tilde{\in} F_{1B} \tilde{\sqsubseteq} F_B} (1 - \tilde{\sigma}(F_{1B}) + S\tilde{\tau}(f_{pu}^{-1}(F_{1B})))$

$$\geq \inf_{f_{pu}(e_M) \tilde{\in} F_{1B} \tilde{\sqsubseteq} F_B} (1 - \tilde{\sigma}(F_B) + S\tilde{\tau}(f_{pu}^{-1}(F_B)))$$

$= SC(f_{pu})$. Hence $\inf_{e_M \tilde{\in} X_A} \inf_{F_B \tilde{\in} SS(Y_B)} \min(1 - N_{f_{pu}(e_M)}(F_B) - SN_{e_M}(f_{pu}^{-1}(F_B))) \geq [f_{pu} \tilde{\in} SC(f_{pu})]$.

Secondly, we prove that $[SC(f_{pu}) \geq SC\alpha_2(f_{pu})]$. From Corollary 3.13, we have $SC(f_{pu}) = \inf_{F_B \tilde{\in} SS(Y_B)} \min(1, 1 - \tilde{\sigma}(F_B) + S\tilde{\tau}(f_{pu}^{-1}(F_B))) = \inf_{F_B \tilde{\in} SS(Y_B)} \min(1, 1 - \inf_{f_{pu}(e_M) \tilde{\in} (F_B)} N_{f_{pu}(e_M)}(F_B) + \inf_{e_M \tilde{\in} f_{pu}^{-1}(F_B)} SN_{e_M}(f_{pu}^{-1}(F_B)))$

$$= \inf_{e_M \tilde{\in} X_A} \min(1, 1 - \inf_{e_M \tilde{\in} f_{pu}^{-1}(F_B)} N_{f_{pu}(e_M)}(F_B) + \inf_{e_M \tilde{\in} f_{pu}^{-1}(F_B)} SN_{e_M}(f_{pu}^{-1}(F_B)))$$

$$\geq \inf_{e_M \tilde{\in} X_A} \inf_{F_B \tilde{\in} SS(Y_B)} \min(1, 1 - N_{f_{pu}(e_M)}(F_B) + SN_{e_M}(f_{pu}^{-1}(F_B))) = SC\alpha_2(f_{pu}).$$

Theorem 4.9 *Let* $(X, \tilde{\tau}, A)$ and $(Y, \tilde{\sigma}, B)$ be two fuzzifying soft topological spaces, $u: X \to Y$ and $p: A \to B$ be mappings. Then

$\vDash f_{pu} \tilde{\in} SC\alpha_2(f_{pu}) \leftrightarrow f_{pu} \tilde{\in} SC\alpha_3(f_{pu})$;

Proof. Clearly from (2) of Theorem 3.14 $\sup_{F_A \tilde{\in} SS(X_A), f_{pu}(F_A) \tilde{\sqsubseteq} F_B} SN_{e_M}(F_A) = \sup_{F_A \tilde{\in} SS(X_A), F_A \tilde{\sqsubseteq} f_{pu}^{-1}(F_B)} SN_{e_M}(F_A)$
$= SN_{e_M}(f_{pu}^{-1}(F_B))$. Then $SC\alpha_3(f_{pu})$
$= \inf_{e_M \tilde{\in} X_A} \inf_{F_B \tilde{\in} SS(Y_B)} \min(1, 1 - N_{f_{pu}(e_M)}(F_B) + \sup_{F_A \tilde{\in} SS(X_A), f_{pu}(F_A) \tilde{\in} (F_B)} SN_{e_M}(F_A))$
$= \inf_{e_M \tilde{\in} X_A} \inf_{F_B \tilde{\in} SS(Y_B)} \min(1, 1 - N_{f_{pu}(e_M)}(F_B) + SN_{e_M}(f_{pu}^{-1}(F_B))) = SC\alpha_2(f_{pu})$.

Lemma 4.10. Let $(X, \tilde{\tau}, A)$ and $(Y, \tilde{\sigma}, B)$ be two fuzzifying soft topological spaces, $u: X \to Y$ and $p: A \to B$ be mappings. Then for any soft sets $F_{iA} \tilde{\in} S(X_A), F_{iB} \tilde{\in} SS(Y_B), i = 1,2$, we have the follwing;

(1) $\vDash F_{1A} \tilde{\sqsubseteq} F_{2A} \to f_{pu}(F_{1A}) \tilde{\sqsubseteq} f_{pu}(F_{2A}), \vDash F_{1A} \equiv F_{2A} \to f_{pu}(F_{1A}) \equiv f_{pu}(F_{1A})$;

(2) $\vDash F_{1B} \tilde{\sqsubseteq} F_{2B} \to f_{pu}^{-1}(F_{1B}) \tilde{\sqsubseteq} f_{pu}^{-1}(F_{2B}), \vDash F_{1B} \equiv F_{2B} \to f_{pu}^{-1}(F_{1B}) \equiv f_{pu}^{-1}(F_{1B})$.

Proof. The proofs are immediate by the truth valuation rules.

Theorem 4.11. Let $(X,\tilde{\tau},A)$ and $(Y,\tilde{\sigma},B)$ be two fuzzifying soft topological spaces, $u: X \to Y$ and $p: A \to B$ be mappings. Then

$\vDash f_{pu} \tilde{\in} SC\alpha_4(f_{pu}) \leftrightarrow f_{pu} \tilde{\in} SC\alpha_5(f_{pu});$

Proof.

Firstly, for any $F_B \tilde{\in} SS(Y_B)$ such that $f_{pu}(F_A) = F_B$ and $F_A \tilde{\sqsubseteq} f_{pu}^{-1}(F_B)$. one deduce that $[semi-Cl_X(f_{pu}^{-1}(F_B)) \tilde{\sqsubseteq} f_{pu}^{-1}(f_{pu}(semi-Cl_X(f_{pu}^{-1}(F_B))))] = 1$, $[semi-Cl_Y(f_{pu}(f_{pu}^{-1}(F_B))) \tilde{\sqsubseteq} semi-Cl_Y(F_B)] = 1$, and $[f_{pu}^{-1}(semi-Cl_Y)(f_{pu}(f_{pu}^{-1}(F_B))) \tilde{\sqsubseteq} f_{pu}^{-1}((semi-Cl_Y(F_B)))] = 1$. Then From (2) of Lemma 4.10 we have $[semi-Cl_X(f_{pu}^{-1}(F_B)) \tilde{\sqsubseteq} f_{pu}^{-1}(Cl_Y(F_B))] \geq [f_{pu}^{-1}(f_{pu}(semi-Cl_X(f_{pu}^{-1}(F_B)))) \tilde{\sqsubseteq} f_{pu}^{-1}(Cl_Y(F_B))] \geq$
$[f_{pu}^{-1}(f_{pu}(semi-Cl_X(f_{pu}^{-1}(F_B)))) \tilde{\sqsubseteq} f_{pu}^{-1}(Cl_Y(f_{pu}(f_{pu}^{-1}(F_B))))] \geq$
$[f_{pu}(semi-Cl_X(f_{pu}^{-1}(F_B))) \tilde{\sqsubseteq} Cl_Y(f_{pu}(f_{pu}^{-1}(F_B)))] \geq [f_{pu}(semi-Cl_X(F_A)) \tilde{\sqsubseteq} Cl_Y(f_{pu}(F_A))]$. Therfore $SC\alpha_5(f_{pu}) = \inf_{F_B \tilde{\in} SS(Y_B)} [semi-Cl_X(f_{pu}^{-1}(F_B)) \tilde{\sqsubseteq} f_{pu}^{-1}(Cl_Y(F_B))] \geq \inf_{F_B \tilde{\in} SS(Y_B)} [f_{pu}(semi-Cl_X(f_{pu}^{-1}(F_B))) \tilde{\sqsubseteq} Cl_Y(f_{pu}(f_{pu}^{-1}(F_B)))]$

$\geq \inf_{F_A \tilde{\in} SS(X_A)} [f_{pu}(semi-Cl_X(F_A)) \tilde{\sqsubseteq} Cl_Y(f_{pu}(F_A))] = SC\alpha_4(f_{pu})$. Secondly, for each $F_A \tilde{\in} SS(X_A)$, there exists $F_B \tilde{\in} SS(Y_B)$ such that $f_{pu}(F_A) = F_B$ and $F_A \tilde{\sqsubseteq} f_{pu}^{-1}(F_B)$. Hence $[semi-Cl_X(f_{pu}^{-1}(F_B)) \tilde{\sqsubseteq} f_{pu}^{-1}(Cl_Y(F_B))] \leq [semi-Cl_X(F_A) \tilde{\sqsubseteq} f_{pu}^{-1}(Cl_Y f_{pu}(F_A))]$. Thus $SC\alpha_4(f_{pu}) = \inf_{F_A \tilde{\in} SS(X_A)} [semi-Cl_X(F_A) \tilde{\sqsubseteq} f_{pu}^{-1}(Cl_Y f_{pu}(F_A))] \geq \inf_{F_B \tilde{\in} SS(Y_B), F_B = f_{pu}(F_A)} [semi-Cl_X(f_{pu}^{-1}(F_B)) \tilde{\sqsubseteq} f_{pu}^{-1}(Cl_Y(F_B))] \geq \inf_{F_B \tilde{\in} SS(Y_B)} [semi-Cl_X(f_{pu}^{-1}(F_B)) \tilde{\sqsubseteq} f_{pu}^{-1}(Cl_Y(F_B))] = SC\alpha_5(f_{pu})$.

Theorem 4.12. Let $(X,\tilde{\tau},A)$ and $(Y,\tilde{\sigma},B)$ be two fuzzifying soft topological spaces, $u: X \to Y$ and $p: A \to B$ be mappings. Then

$\vDash f_{pu} \tilde{\in} SC\alpha_5(f_{pu}) \leftrightarrow f_{pu} \tilde{\in} SC\alpha_2(f_{pu});$

Proof.

$$\begin{aligned}
SC\alpha_5(f_{pu}) &= [\forall F_B(semi-Cl_X(f_{pu}^{-1}(F_B)) \tilde{\sqsubseteq} f_{pu}^{-1}(Cl_Y(F_B)))] \\
&= \inf_{F_B \tilde{\in} SS(Y_B)} \inf_{e_M \tilde{\in} X_A} \min(1, 1-(1-SN_{(e_M)}(X_A \backslash (f_{pu}^{-1}(F_B)))) + 1 - N_{f_{pu}(e_M)}(Y_B \backslash (F_B))) \\
&= \inf_{F_B \tilde{\in} SS(Y_B)} \inf_{e_M \tilde{\in} X_A} \min(1, 1 - N_{f_{pu}(e_M)}(Y_B \backslash F_B) + SN_{(e_M)}(X_A \backslash f_{pu}^{-1}(F_B))) \\
&= \inf_{F_{1B} \tilde{\in} SS(Y_B)} \inf_{e_M \tilde{\in} X_A} \min(1, 1 - N_{f_{pu}(e_M)}(F_{1B}) + SN_{(e_M)}(f_{pu}^{-1}(F_{1B}))) = SC\alpha_2(f_{pu}).
\end{aligned}$$

Theorem 4.13. Let $(X,\tilde{\tau},A)$ and $(Y,\tilde{\sigma},B)$ be two fuzzifying soft topological spaces, $u: X \to Y$ and $p: A \to B$ be mappings. Then

$\vDash f_{pu} \tilde{\in} SC(f_{pu}) \leftrightarrow f_{pu} \tilde{\in} SC\alpha_j(f_{pu}), \quad j = 1,2,3,4,5;$

Proof. It can be obtain from Theorem 4.7, Theorem 4.8, Theorem 4.9, Theorem 4.11 and Theorem4.12.

5. Conclusion

By applying a new approach to concept of both soft belonging which is called soft element and two distinct soft elements ,we success to introduce and study the concepts of soft semi open sets and soft semi continuity in fuzzifying soft topological spaces which are defined over an initial universe with a fixed set of parameters. In future, these results may be extended to new types of soft fuzzifying generalized closed and open sets in soft fuzzifying topological spaces.

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