On the $B(r,s,t,u)$ Matrix Domain in the Sequence Space $\mathcal{L}_u$

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Abstract: In this paper, I introduce a new sequence space $B(\mathcal{L}_u)$ as the domain of four dimensional generalized difference matrix $B(r,s,t,u)$ and as a generalization of the series space BV. I give some topological properties with some inclusion relations. Moreover, I calculate beta(bp)- and gamma- duals of the new sequence space $B(\mathcal{L}_u)$. In the last section I characterize some matrix classes and I conclude the paper with some important results.

Keywords: Generalized Difference Matrix, Matrix Domain, Matrix Transformations, Dual of Sequence Spaces

1. Introduction

We denote the space of all real or complex valued double sequences by Ω which is a vector space with coordinatewise addition and scalar multiplication. Each vector subspace of Ω is called as a double sequence space. The spaces of all bounded double sequences are denoted by M_u which is a Banach space with the supremum norm $||.||_\infty$. Any double sequence $x = (x_{kl})$ is said to be convergent in Pringsheim's sense to the limit point $l \in \mathbb{C}$ if $p - \lim_{k,l\to\infty} x_{kl} = l$; where $\mathbb{C}$ denotes the complex field. The set of all convergent double sequences in Pringsheim's sense is denoted by C_p. Although every convergent single sequence is bounded, convergence double sequences do not guarantee the boundedness in general. That is, there exists some double sequences which is bounded bot not convergent, i.e., the set $M_u \backslash C_p$ is not empty. We may consider the set of all convergent in pringsheim's sense and bounded double sequences by C_{bp}, that is $C_{bp} = C_p \cap M_u$. A double sequence $x = (x_{kl})$ is called regular convergent if it is a single convergent sequence with respect to each index. We may denote the space of all such sequences by C_r. Moreover, by C_{r0} and C_{bp0} we denote the set of all null double sequences included in the double sequence space C_r and C_{bp}, respectively. The set of all absolutely q −summable double sequences was defined by Başar and Sever (2009) as,

$$\mathcal{L}_q = \left\{ x = (x_{kl}) \in \Omega: \sum_{k,l} |x_{kl}|^q < \infty \right\}, (1 \leq q < \infty)$$

which is a Banach space with the norm $||.||_q$. The space $\mathcal{L}_u$ as a special case of $\mathcal{L}_q$ with $q = 1$ was introduced by Zeltser (2002). The set of double sequences of bounded variation BV whose sequences of partial sums are convergent in Pringsheim's sense was defined by Altay and Başar (2005) as follow,

$$BV = \left\{ x = (x_{kl}) \in \Omega : \sum_{k,l} \left| x_{kl} - x_{k-1,l} - x_{k,l-1} + x_{k-1,l-1} \right| < \infty \right\}.$$

which is a Banach space with the norm $\|.\|_{BV}$.

The linear convergence rule for a double sequence space λ with respect to ϑ is defined by $\vartheta - lim: \lambda \to \mathbb{C}$. The sum of a double series $\sum_{k,l} x_{kl}$ related to this rule is shown by $\vartheta - \sum_{k,l} x_{kl} = \vartheta - \lim_{m,n\to\infty} \sum_{k,l=0}^{m,n} x_{kl}$. Throughout the paper we consider that ϑ denotes any of the symbols from the set $\{p, bp, r\}$.

The $\alpha -$dual of any double sequence space λ, $\beta(\vartheta) -$dual of any double sequence space λ with respect to the $\vartheta -$convergence and γ-dual of any double sequence space λ respectively defined as

$$\lambda^{\alpha} = \left\{ a = (a_{kl}) \in \Omega : \sum_{k,l} |a_{kl} x_{kl}| < \infty \; for \; all \; x = (x_{kl}) \in \lambda \right\}$$

$$\lambda^{\beta(\vartheta)} = \left\{ a = (a_{kl}) \in \Omega : \vartheta - \sum_{k,l} a_{kl} x_{kl} \; exists \; for \; all \; x = (x_{kl}) \in \lambda \right\}$$

$$\lambda^{\gamma} = \left\{ a = (a_{kl}) \in \Omega : \sup_{m,n \in \mathbb{N}} \left| \sum_{k,l=0}^{m,n} a_{kl} x_{kl} \right| < \infty \; for \; all \; x = (x_{kl}) \in \lambda \right\}$$

For any two double sequence spaces λ and μ, we can easily say that $\mu^{\alpha} \subset \lambda^{\alpha}$ whenever $\lambda \subset \mu$ and $\lambda^{\alpha} \subset \lambda^{\gamma}$. It is known that the $\vartheta -$convergence of double sequence of the partial sum of a double series does not guarantee its boundedness. Thus, the inclusion $\lambda^{\alpha} \subset \lambda^{\beta(\vartheta)}$ holds while $\lambda^{\beta(\vartheta)} \subset \lambda^{\gamma}$ does not hold.

Let λ and μ be two double sequence spaces, and $A = (a_{mnkl})$ be any four dimensional infinite matrix of real or complex numbers, where $m, n, k, l \in \mathbb{N}$. For every sequence $x = (x_{kl}) \in \lambda$ the sequence $Ax = ((Ax)_{mn}) \in \mu$ is called $A -$transform of x, where

$$(Ax)_{mn} = \vartheta - \sum_{k,l=0}^{\infty} a_{mnkl}\, x_{kl} \, , for \; each \; m, n \in \mathbb{N} \qquad (1.1)$$

Then, A defines a four dimensional matrix mapping from λ to μ and we show it by writing $A: \lambda \longrightarrow \mu$. We may define $\vartheta -$summability domain $\lambda_A^{(\vartheta)}$ of A in the space λ of double sequences $x = (x_{kl})$ as,

$$\lambda_A^{(\vartheta)} = \left\{ x = (x_{kl}) \in \Omega : \mathrm{Ax} = \left(\vartheta - \sum_{k,l=0}^{\infty} a_{mnkl}\, x_{kl} \right)_{m,n \in \mathbb{N}} \text{exists and is in } \lambda \right\}$$

By $A = (a_{mnkl}) \in (\lambda : \mu)$, we denote the set of all four dimensional matrices A such that $A : \lambda \longrightarrow \mu$ if and only if the series on the right side of (1.1) converges for each $m, n \in \mathbb{N}$ in the sense of ϑ for every $x \in \lambda$, and we have $Ax = ((Ax)_{mn})$ belongs to μ for all $x \in \lambda$. A sequence x is said to be A-summable to l and is called as the A-limit of x. Adams, C. R. (1933) defined that a four dimensional infinite matrix $A = (a_{mnkl})$ is called triangular if $a_{mnkl} = 0$ for $k > m$ or $l > n$ or both and Cooke (1950) showed that every triangular matrix has a unique inverse which is also triangular.

Recently, Tuğ (2017) has defined and studied the four dimensional generalized difference matrix and its domain on some double sequence spaces. The four dimensional matrix $B(r,s,t,u) = \{b_{mnkl}(r,s,t,u)\}$ was defined for all $r,s,t,u \in \mathbb{R}\backslash\{0\}$ as

$$b_{mnkl}(r,s,t,u) = \begin{cases} su, & (k,l) = (m-1,n-1) \\ st, & (k,l) = (m-1,n) \\ ru, & (k,l) = (m,n-1) \\ rt, & (k,l) = (m,n) \\ 0, & otherwise \end{cases}$$

for all $m,n,k,l \in \mathbb{N}$. The $B-$transform of any double sequence $x = (x_{kl})$ was given by

$$y_{mn} = \{B(r,s,t,u)x\}_{mn} = \sum_{kl} b_{mnkl}(r,s,t,u)x_{kl} \tag{1.2}$$

$$= sux_{m-1,n-1} + stx_{m-1,n} + rux_{m,n-1} + rtx_{mn}$$

for all $m,n \in \mathbb{N}$. The inverse matrix $B^{-1}(r,s,t,u) = F(r,s,t,u) = \{f_{mnkl}(r,s,t,u)\}$ was shown by,

$$f_{mnkl}(r,s,t,u) = \begin{cases} \dfrac{(\frac{-s}{r})^{m-k}(\frac{-u}{t})^{n-l}}{rt}, & 0 \le k \le m, 0 \le l \le n \\ 0 & , \quad otherwise \end{cases}$$

for all $m,n,k,l \in \mathbb{N}$. One can obtain by applying the inverse matrix $F(r,s,t,u)$ to (1.2) that

$$x_{mn} = \frac{1}{rt}\sum_{k,l=0}^{m,n}\left(\frac{-s}{r}\right)^{m-k}\left(\frac{-u}{t}\right)^{n-l} y_{kl}, \qquad for\ all\ m,n \in \mathbb{N}. \tag{1.3}$$

From now on, we suppose that the terms of double sequences $x = (x_{mn})$ and $y = (y_{mn})$ are connected with the relation (1.2). It is obviously seen from the $B-$transform of any sequence $x = (x_{mn})$ will be considered as $\Delta = B(1,-1,1,-1)-$transform in the case $r = t = 1$ and $s = u = -1$ for all $m,n \in \mathbb{N}$. Note that the results of $B(r,s,t,u)$ domain of double sequences are more general than the Δ domain of double sequence spaces.

2. The New Double Sequence Space $B(\mathcal{L}_u)$

In this section, the new sequence space $B(\mathcal{L}_u)$ is defined as four dimensional matrix $B(r,s,t,u)$ domain in the sequence space $\mathcal{L}_u$.

$$B(\mathcal{L}_u) = \left\{x = (x_{kl}) \in \Omega: \sum_{k,l}\left|sux_{k-1,l-1} + stx_{k-1,l} + rux_{k,l-1} + rtx_{kl}\right| < \infty\right\}.$$

The sequence space $B(\mathcal{L}_u)$ is reduced to the double sequence space BV in the case $r = t = 1$ and

$s = u = -1$. So the new sequence space $B(\mathcal{L}_u)$ is called a generalization of the double sequence space BV and the following theorems can be given.

Theorem 2.1. The double sequence space $B(\mathcal{L}_u)$ is a linear space with coordinate wise addition and scalar multiplication and is a Banach space with the norm

$$\|x\|_{B(\mathcal{L}_u)} = \sum_{k,l} |sux_{k-1,l-1} + stx_{k-1,l} + rux_{k,l-1} + rtx_{kl}|$$

which is norm isomorphic to the space $\mathcal{L}_u$.

Proof. The linearity of the space is easy to prove so we omit the details. Now let consider a Cauch sequence $x^i = \left\{x_{kl}^{(i)}\right\}_{k,l\in\mathbb{N}}$ for every fixed $i \in \mathbb{N}$ in the space $B(\mathcal{L}_u)$. Then we can write here that for a given $\varepsilon > 0$ there exist a positive real number $M(\varepsilon)$ such that

$$\|x^i - x^j\|_{B(\mathcal{L}_u)} = \sum_{k,l} \left|\{B(r,s,t,u)x^i\}_{kl} - \{B(r,s,t,u)x^j\}_{kl}\right| < \varepsilon$$

for all $k, l \geq M(\varepsilon)$. Then obviously we can say that $\left\{\{B(r,s,t,u)x^i\}_{kl}\right\}_{i\in\mathbb{N}}$ is a Cauchy sequence $\mathcal{L}_u$ for each fixed $k, l \in \mathbb{N}$. Since the double sequence space $\mathcal{L}_u$ is complete, every Cauchy sequence is convergent in $\mathcal{L}_u$as i tends to infinity. That is there exists a sequence $\{B(r,s,t,u)x\} \in \mathcal{L}_u$ such that $\{B(r,s,t,u)x^i\}_{kl} \to \{B(r,s,t,u)x\} \; as \; i \to \infty$ which says that for a given $\varepsilon > 0$ we have the following inequality that,

$$\left|\{B(r,s,t,u)x^i\}_{kl} - \{B(r,s,t,u)x\}_{kl}\right| < \varepsilon$$

for all $k, l \in \mathbb{N}$. Moreover, because $\{B(r,s,t,u)x^i\}_{kl} \in \mathcal{L}_u$ for each fixed $i \in \mathbb{N}$, there exists a positive real number K such that $\sum_{k,l} \left|\{B(r,s,t,u)x^i\}_{kl}\right| \leq K$. With the above results, we have power to write the following inequality that

$$\begin{aligned}\sum_{k,l} |\{B(r,s,t,u)x\}_{kl}| &\leq \sum_{k,l} \left|\{B(r,s,t,u)x\}_{kl} + \{B(r,s,t,u)x^i\}_{kl} - \{B(r,s,t,u)x^i\}_{kl}\right| \\ &\leq \sum_{k,l} \left|\{B(r,s,t,u)x^i\}_{kl} - \{B(r,s,t,u)x\}_{kl}\right| + \sum_{k,l} \left|\{B(r,s,t,u)x^i\}_{kl}\right| \\ &< \varepsilon + K.\end{aligned}$$

The last approach means that $B(r,s,t,u)x \in \mathcal{L}_u$, i.e., $x \in B(\mathcal{L}_u)$. By this result, it has been shown that the double sequence space $B(\mathcal{L}_u)$ is a complete normed space with the given norm.

Now we should show that the spaces $B(\mathcal{L}_u)$ and $\mathcal{L}_u$ are norm isomorphic with a transform from $B(\mathcal{L}_u)$ to $\mathcal{L}_u$. Let consider this transform T, by $x \to Tx = y = B(r,s,t,u)x$. Then it can be easily proved that T is injective. Moreover by using the relation (1.2) between the double sequences $x = (x_{kl})$ and $y = (y_{kl})$ we have the following equality that

$$|\{B(r,s,t,u)x\}_{kl}| = |y_{kl}|$$

which gives us that $\|\{B(r,s,t,u)x\}_{kl}\|_{B(\mathcal{L}_u)} = \|y\|_q$. Thus, T is surjective. This is what we expected in the theorem.

Theorem 2.2. Suppose that $\vartheta \in \{p, bp, r\}$ and $r = t = 1$ and $s = u = -1$. Then, the inclusions $B(\mathcal{L}_u) \subset C_\vartheta$ and $B(\mathcal{L}_u) \subset M_u$ strictly hold.

Proof. Let $r = t = 1$ and $s = u = -1$ and $x = (x_{kl}) \in B(\mathcal{L}_u)$. Then we have

$$\sum_{k,l} |\{B(r,s,t,u)x\}_{kl}| < \infty,$$

that is the series $\sum_{k,l}(\{B(r,s,t,u)x\}_{kl})$ is convergent. Since we may write this sequence in the form

$$x_{mn} = \sum_{k,l=0}^{m,n} (\{B(r,s,t,u)x\}_{kl})$$

which is convergent, says that $x \in C_\vartheta$. In order to prove the inclusion and $B(\mathcal{L}_u) \subset M_u$ hold, let us take the following inequality that

$$\sup_{m,n\in\mathbb{N}} |x_{mn}| = \sup_{m,n\in\mathbb{N}} \left|\sum_{k,l=0}^{m,n} (\{B(r,s,t,u)x\}_{kl})\right|$$

$$\leq \sup_{m,n\in\mathbb{N}} \sum_{k,l=0}^{m,n} |\{B(r,s,t,u)x\}_{kl}| < \infty$$

means that $x \in M_u$. Furthermore, if we define a double sequence $x = (x_{kl})$ as in the following

$$x_{kl} = \begin{cases} \dfrac{2}{l+2}, & k = 0 \text{ } and \text{ } l \text{ } is \text{ } even \\ o \quad , & otherwise \end{cases}$$

for all $k, l \in \mathbb{N}$. Obviously it can be shown that $x \in C_\vartheta$ and $x \in M_u$ but not in the sequence space $B(\mathcal{L}_u)$. This proves that the inclusions are strict.

Theorem 2.3. Let $\left(\frac{-s}{r}\right) > 1$ and $\left(\frac{-u}{t}\right) > 1$. Then the inclusion $\mathcal{L}_u \subset B(\mathcal{L}_u)$ strictly holds.

Proof. It can be easily proved by choosing a double sequence $x = (x_{kl})$ as

$$x_{kl} = \left(\frac{-s}{r}\right)^k \left(\frac{-u}{t}\right)^l$$

and by having B −transform of this double sequence $x = (x_{kl})$ as $\{B(r,s,t,u)x\}_{kl} = 0$. So $x \in B(\mathcal{L}_u)$ but not in $\mathcal{L}_u$. We pass the details.

3. $\boldsymbol{\beta(bp)}$ − and $\boldsymbol{\gamma}$ −Dual of the Space $\boldsymbol{B(\mathcal{L}_u)}$

In this section, I calculate the $\beta(bp)$ −dual and γ −dual of the new double sequence space $B(\mathcal{L}_u)$ after giving some related lemmas.

Lemma 3.1. (Yeşilkayagil & Başar, 2017)**.** Suppose that $A = (a_{mnkl})$ be a four dimensional matrix. Then $A \in (\mathcal{L}_u : C_{bp})$ if and only if

(3.1.1) $$\sup_{m,n,k,l\in\mathbb{N}} |a_{mnkl}| < \infty$$

(3.1.2) $$\exists a_{kl} \in \mathbb{C}, \ni \lim_{m,n\to\infty} a_{mnkl} = a_{kl} \;\; for\ all\ k,l \in \mathbb{N}$$

Lemma 3.2. (Yeşilkayagil & Başar, 2017). Suppose that $A = (a_{mnkl})$ be a four dimensional matrix. Then $A \in (\mathcal{L}_u : M_u)$ if and only if the condition (3.1.1) holds.

Theorem 3.3. Let us define the sets β_1 and β_2 as in the following.

$$\beta_1 = \left\{ a = (a_{ji}) \in \Omega : \exists \beta_{kl} \in \mathbb{C}, \ni \lim_{m,n\to\infty} \sum_{j,i=k,l}^{m,n} \left(\frac{-s}{r}\right)^{j-k} \left(\frac{-u}{t}\right)^{i-l} \frac{a_{ji}}{rt} = \beta_{kl} \;\; for\ all\ k,l \in \mathbb{N} \right\}$$

$$\beta_2 = \left\{ a = (a_{ji}) \in \Omega : \sup_{min\in\mathbb{N}} \left| \sum_{j,i=k,l}^{m,n} \left(\frac{-s}{r}\right)^{j-k} \left(\frac{-u}{t}\right)^{i-l} \frac{a_{ji}}{rt} \right| < \infty \right\}$$

Then the $\beta(bp)$ −dual of the space $B(\mathcal{L}_u)$ is the set $\beta_1 \cap \beta_2$. That is, $\{B(\mathcal{L}_u)\}^{\beta(bp)} = \beta_1 \cap \beta_2$.

Proof. Let us suppose that $a = (a_{kl}) \in \Omega$ and $x = (x_{kl}) \in B(\mathcal{L}_u)$. Then, we have the following equality that

$$\sum_{k,l=0}^{m,n} a_{kl} x_{kl} = \sum_{k,l=0}^{m,n} a_{kl} \sum_{j,i=0}^{m,n} \left(\frac{-s}{r}\right)^{j-k} \left(\frac{-u}{t}\right)^{i-l} \frac{y_{ji}}{rt}$$

$$= \sum_{k,l=0}^{m,n} \sum_{j,i=k,l}^{m,n} \left(\frac{-s}{r}\right)^{j-k} \left(\frac{-u}{t}\right)^{i-l} \frac{a_{ji}}{rt} y_{kl}$$

$$= (Dy)_{mn}$$

Where the four dimensional matrix $D = (d_{mnkl})$ defined as

$$d_{mnkl} = \begin{cases} \sum_{j,i=k,l}^{m,n} \left(\frac{-s}{r}\right)^{j-k} \left(\frac{-u}{t}\right)^{i-l} \frac{a_{ji}}{rt}, & o \le k \le m, 0 \le l \le n \\ 0 \qquad\qquad , & Otherwise \end{cases}$$

$for\ all\ m,n,k,l \in \mathbb{N}$**.** Now we may say that the $(m,n)th$ partial sum of the series $\sum_{k,l} a_{kl} x_{kl}$ is in the space CS_{bp} whenever $x = (x_{kl}) \in B(\mathcal{L}_u)$ if and only if $Dy \in C_{bp}$ whenever $y = (y_{kl}) \in \mathcal{L}_u$. The last conclusion gives us that $a = (a_{kl}) \in \{B(\mathcal{L}_u)\}^{\beta(bp)}$ if and only if $D \in (\mathcal{L}_u : C_{bp})$. Thus, we can obtain from the Lemma 3.1. that the conditions (3.1.1) and (3.1.2) hold with the four dimensional matrix $D = (d_{mnkl})$ instead of the matrix $A = (a_{mnkl})$. That is

$\exists \beta_{kl} \in \mathbb{C}, \ni \ \lim_{m,n\to\infty} \sum_{j,i=k,l}^{m,n} \left(\frac{-s}{r}\right)^{j-k} \left(\frac{-u}{t}\right)^{i-l} \frac{a_{ji}}{rt} = \beta_{kl} \ \ for\ all\ k,l \in \mathbb{N}$, and

$\sup_{min\in\mathbb{N}} \left| \sum_{j,i=k,l}^{m,n} \left(\frac{-s}{r}\right)^{j-k} \left(\frac{-u}{t}\right)^{i-l} \frac{a_{ji}}{rt} \right| < \infty,$

satisfied. The last approaches give us that $\{B(\mathcal{L}_u)\}^{\beta(bp)} = \beta_1 \cap \beta_2$.

Theorem 3.4. The γ −dual of the space $B(\mathcal{L}_u)$is the set β_2, that is, $\{B(\mathcal{L}_u)\}^{\gamma} = \beta_2$

Proof. It can be proved in the same method of the Theorem 3.3. by having the $(m,n)th$ partial sum of the series $\sum_{k,l} a_{kl}x_{kl}$ is in the space BS whenever $x = (x_{kl}) \in B(\mathcal{L}_u)$ if and only if $Dy \in M_u$ whenever $y = (y_{kl}) \in \mathcal{L}_u$. The last conclusion gives us that $a = (a_{kl}) \in \{B(\mathcal{L}_u)\}^{\gamma}$ if and only if $D \in (\mathcal{L}_u: M_u)$. Thus, we can obtain from the Lemma 3.1. that the condition (3.1.1) holds with the four dimensional matrix $D = (d_{mnkl})$ instead of the matrix $A = (a_{mnkl})$. That is,

$\sup_{min\in\mathbb{N}} \left| \sum_{j,i=k,l}^{m,n} \left(\frac{-s}{r}\right)^{j-k} \left(\frac{-u}{t}\right)^{i-l} \frac{a_{ji}}{rt} \right| < \infty,$

satisfied. Which gives us that $\{B(\mathcal{L}_u)\}^{\gamma} = \beta_2$. So, we omit the details.

4. Matrix Transformations Related with the Double Sequence Space $B(\mathcal{L}_u)$

In this section, I characterize some new matrix classes which are related with the double sequence space $B(\mathcal{L}_u)$ and I conclude the paper with some related results.

Theorem 4.1. The four dimensional matrix $A = (a_{mnkl}) \in (\mathcal{L}_u: B(C_{bp}))$ if and only if the following conditions hold.

(4.1.1) $$\sup_{m,n,k,l\in\mathbb{N}} \left| \sum_{j,i=0}^{m,n} b_{mnji}(r,s,t,u)\, a_{jikl} \right| < \infty$$

(4.1.2) $$\exists a_{kl} \in \mathbb{C}, \ni \lim_{m,n\to\infty} \sum_{j,i=0}^{m,n} b_{mnji}(r,s,t,u)\, a_{jikl} = a_{kl} \ \ for\ all\ k,l \in \mathbb{N}$$

Theorem 4.2. The four dimensional matrix $A = (a_{mnkl}) \in (\mathcal{L}_u: B(M_u))$ if and only if the condition (4.1.1) hold.

Corollary 4.3. Let $A = (a_{mnkl})$ be a four dimensional infinite matrix. Then the following statements are satisfied.

i) $A = (a_{mnkl}) \in (B(\mathcal{L}_u): B(M_u))$ if and only if the condition (3.1.1) holds with h_{mnkl} instead of a_{mnkl} where $H = (h_{mnkl}) = B(AF(r,s,t,u))$

ii) $A = (a_{mnkl}) \in (B(\mathcal{L}_u): B(C_{bp}))$ if and only if the conditions (3.1.1) and (3.1.2) hold with h_{mnkl} instead of a_{mnkl} where $H = (h_{mnkl}) = B(AF(r,s,t,u))$

5. Conclusion

The matrix domain in the sequence spaces is one of the highlighted content in Functional analysis. Especially, special triangular matrices like Nörlund matrix, Cesero matrix, Riesz mean, etc. domains in the single and double sequence spaces are the most studied methods nowadays. Matrix domain of any sequence space is larger than the original space. Therefore, by calculating the matrix domain of any sequence space, we have a new set of sequences whose original set is the subset of the new generated.

In this paper, I tried to fill the gap in the literature for the sequence space $\mathcal{L}_u$ and its B-transform $B(\mathcal{L}_u)$. Recently the four dimensional generalized difference matrix $B(r,s,t,u)$ domain of some double sequences was defined studied by Tuğ (2017). As a natural continuation of Tuğ (2017), I tried to complete some important missing part of the literature. Moreover I tried to extend the sequence space BV with respect to the $B(r,s,t,u)$ domain of the double sequence space $\mathcal{L}_u$. I proved some topological properties with some inclusion relations and I calculated $\beta(bp)$ −dual and γ −dual of the new double sequence space $B(\mathcal{L}_u)$. Moreover, I characterized some new four dimensional matrix classes related with the sequence space $B(\mathcal{L}_u)$. One can easily obtained from the conclusion of this paper that the new matrix classes $(B(\mathcal{L}_u): C_\vartheta)$, $(B(\mathcal{L}_u): B(C_\vartheta))$ and $(\mathcal{L}_u: B(C_\vartheta))$ where ϑ is any of the sign form the set $\{p, bp, r, f\}$ are still open problems for the readers.

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