Investigation of a Developed Deflection Control Model of Reinforced Concrete Two Way Slab Systems

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Abstract: A model was presented in a previous study which provided a rational approach to deflection control of reinforced concrete two way slabs considering uncertainties in structural behavior and deflection limits was investigated thoroughly in this work. A simulation model taking into account the uncertainties in materials and loads along with sensitivity analysis of results was developed. Results of simulation represented in the form of probability density function (PDF) and cumulative density function (CDF), optimum thickness and results of sensitivity analysis of reinforced concrete two-way slab systems were presented. In this study, uncertainties in time effects (Creep and shrinkage) are taken into account by using Monte Carlo simulation and are based on proposed variable parameters taken from major references. In general, the proposed procedure results in smaller thicknesses for two-way slab-beam system than ACI 318-14 recommendations for longer spans and larger thicknesses for shorter spans.

Keywords: Deflection Control, Two-Way Slabs, Monte-Carlo Simulation, Sensitivity Analysis

1. Introduction

In a very recent paper, Al-Nu’man and Abdullah (2018) developed a serviceability model based on utility theory and sensitivity analysis for two way reinforced concrete slab systems in particular. This paper investigates the developed model through a comparative study of notable deflection investigations found in the literature**.** A methodology based on the application of utility theory in combination with probabilistic analyses was proposed. This approach recognizes that serviceability failure is an economic issue since an unserviceable structure may have an adequate margin of safety against collapse. A simplified procedure was presented for estimating optimum structural parameter (member depth) by the applicability of utility theory as a basis for developing deflection control criteria. Monte Carlo simulation is used to develop histograms of selected deflection parameter; a serviceability loss function is then specified to define the onset of serviceability failure and an upper limit representing complete serviceability failure with associate costs. Optimum slab thickness is obtained by minimizing total cost consisting of initial construction cost and probabilistic cost of failure. Results for two-way slabs are developed and compared with ACI code provision for minimum thickness.

This study would cover the optimum proportioning of two-way floor systems which is not found in literature. Also, this study considers the local practice and associated costs and develops accordingly a rational proportioning model based on utility theory. Moreover, this model brings an opportunity to compare with codes recommendations (ACI in particular). This would point out the stand of the model as a rational economic –oriented model more than what would be a criticism to the codes.

2. Concept of Monte Carlo Simulation

Monte Carlo simulation is a numerical method used to find solutions to mathematical problems using random numbers. Often, the method is used when the problem involves uncertainty, a large number of variables, or nonlinearities, or other features which make it difficult to solve analytically. The method becomes more efficient compared to other numerical methods as the dimension of the problem increases.

The basis of Monte Carlo simulation is experimentation on the chance (or probabilistic) elements through random sampling. The technique breaks down into five simple steps (Al-Nu'man & Abdullah, 2018; Barry, Ralfh, & Hanna, 2003):

1- Setting up a probability distribution for important variables.

2- Building a cumulative probability distribution for each variable.

3- Establishing an interval of random number for each variable.

4- Generating random numbers.

5- Actually simulating a series of trials.

Results can be presented as cumulative or frequency distributions which clearly communicate range of possible outcomes.

3. Concept of Utility Theory

The basic idea of serviceability design based on utility theory is that structural service ability can be considered as a type of utility defined in monetary terms as the difference between the benefit obtained from a serviceable structure minus the total cost consisting of initial cost of construction minus cost of serviceability failure. If the benefit obtained from a serviceable structure is considered a constant, the maximum utility is obtained when the total cost is a minimum. The cost of construction can be calculated in the usual fashion by computing quantities and applying appropriate unit prices. To calculate the cost of failure consideration must be given to the variability of deflection response for a given member and the fact that serviceability in general does not have a crisp limit. For a given member, the deflection response can be represented by a probability distribution. Superimposed on the distribution is a loss function that defines the onset of serviceability failure up to complete serviceability failure; i.e. the deflection at which the structure becomes completely unserviceable with an associated cost to remedy (Rosowsky & Stewart, 2001; Hossain & Stewart, 2001). The greater the overlap between the loss function and the distribution, the greater the probabilistic cost of failure.

4. Serviceability Loss Function

It is understood that unservice ability due to excessive deflection reduces property values and rental income as well as causing disturbance to existing occupants (which produces intangible costs). Non-structural repairs may be quite costly, and in most cases, such repairs do not provide any permanent solution to time-dependent deflection induced problems. Hence, total deflection will continue to increase (due to creep and shrinkage effects); and ultimately, structural repair or replacement is the only option to reactivate the utility of the structure. Such remedial actions are likely to be expensive compared to individual non-structural expenditures for unservice ability.

In the present study, the data presented by Hossain and Stewart, (2001) are used to establish upper and lower bound for continuous loss function for direct cost of repair. For convenience, the Cumulative density function (CDF) is used to define the loss function (Black & Fredrric, 1985) between the two limits because it is assumed in this study that expected repair costs follow the probability of damaging deflection.

5. Optimum Thicknesses of Two-Way Reinforced Concrete Slab Systems

This section is divided into two parts, part one includes proposed model for the estimation of optimum thicknesses of two-way reinforced concrete slab systems. Part two includes comparative study between calculated and measured long term deflections and optimum thickness obtained according to the results of sensitivity analysis of reinforced concrete slab systems which are explained in the following chapter.

6. Optimum Thickness Determination

The first stage is the determinations of immediate and long-term deflections taking into account the uncertainties due to member behavior and loading by using Monte Carlo Simulation. The second is to determine of probabilities of failure of slabs, and superimposing the Probability Density Function (PDF) (obtained from simulation results) and serviceability loss function, followed by estimating optimum thicknesses of reinforced concrete two way slabs. A sensitivity analysis is then made to see how is the sensitivity of the results to variations in assumed loss functions and assumed costs. The models of two-way slabs are applied to interior panels in a general multistory structure; three types of floor system are used in this study, flat plate, and flat slab, and slab-beam systems. The size and thickness of these models are as follows:

For flat plates and slab-beam systems, the clear span size is 6 *6m and thicknesses are 130, 140, 150, 160, 170, 180, 190, 200, 210, 220mm. For flat slabs, the clear span size is 7.5*7.5m and thicknesses are 180, 190, 200, 210, 220, 230, 240mm with drop panels thickness equals to 0.25*thickness of slab.

In this study, a simulation model represented by Monte Carlo Simulation is used for deflections (initial & long-term) predicted according to ACI 209 Recommendations (ACI committee 209, 2008) approach. The general equation of ultimate deflection is summation of deflection due to (dead load, creep, shrinkage, and live load) respectively:

7. Sources of Uncertainty and Variability

Additional uncertainties in predicting deflections by calculations occur due to modeling errors in the calculation procedure including treatment of boundary conditions, the effects of restraint stresses on crack development, and uncertainties in the time of installation of nonstructural elements when considering incremental deflections occurring after the installation of nonstructural elements. This study focuses on the effect of variability of member properties and loads. Assumed statistical properties of the variables considered are listed in Table 1.

Table 1: Probability model of random variables (Al-Nu'man & Abdullah, 2018)

Variable		Mean	COV	S.D	Reference
Concrete (in place)	$f'c$ (MPa)	$0.675f'c+7.58 \leq 1.15 f'c$	0.176	□	(Scanlon et al, 2007)
	fr (MPa)	$0.69\sqrt{f'_c}$	0.218	□	(Scanlon et al, 2007)
	Ec (MPa)	$4700\sqrt{f'_c}$	-	□	(ACI committee 318, 2014)
Reinforcement	As	0.99 An	0.024	□	(Scanlon et al, 2007)
	Es (MPa)	201326.91	0.024	□	(Rosowsky and Stewart, 2001)
Creep	νu	2.35	□	0.6	(ACI committee 209, 2008)
	ψ	0.6	□	6.66*10-2	(ACI committee 209, 2008)
Shrinkage	d	10 days	□	6.66 days	(Choi et al, 2004)
	(εsh) u	780*10-6	□	121.6*10-6	(Choi et al, 2004)
	α	1.0	□	3.33*10-2	(Choi et al, 2004)
	f	55 days	□	25 days	(Choi et al, 2004)

8. Construction Load and (Load Time Histories)

Long-time deflections depend on the load-time history. It is assumed that the critical deflection is strongly related to a load-time history, designated LH1. Table 2 shows the probability of assumed load history. Generally, the loads acting on the office floor during its construction and use are:

Sustained dead load: self-weight of the structural members.
Sustained construction live load: weight of construction workers, construction equipment, and stacking of materials after concrete placement.
Sustained floor live loads: those loads are relatively constant within a particular occupancy; namely, the weight of people, furniture, partition, and other portable fixtures and equipment.

Table 2: Probability load model of random variables (Al-Nu'man & Abdullah, 2018)

Load		Statistical parameters	Distribution	Reference
Construction load	Form work load(additional dead load)	Mean =0.11 Dn COV= 0.10	Normal	(El-Shahhat et al, 1993)
	Sustained construction live load	Mean=0.29kPa COV= 1.10	Normal	(Ayoub and Karshenas, 1994)
	Stacking load	Mean=0.974kPa COV= 0.60	Normal	(Ayoub and Karshenas, 1994)
Dead load		Mean =1.05 Dn COV= 0.10	Normal	(Stewart, 1996)
Live load	Sustained live load	Mean =2.4kPa COV=0.056	Normal	(Ellinwood and Culver, 1977)

9. Calculation of Optimum Thickness

Determination of optimum slab thickness is dependent in this study on the optimum cost, which consists of construction cost plus cost of failure. Failure cost equals product of (summation of direct cost (repair cost) and indirect cost (loss of product)), by λ (percentage of failure) obtained by superimposing Probability Density Function (PDF) and Serviceability loss function. After that the optimum slab thickness is selected as for slabs that have minimum cost (considering the deflection control and taking into account the effect of creep, and shrinkage in floor).

10. Comparative Study

In this part, a comparative study is made between calculated and measured long-term deflections and slab thickness for interior panels of six multistory buildings reported by many authors such as Taylor[24], Heiman[24], Jokinen[25], Sbarounis[26], Rangan[27] and Jawad[28]. These are fully documented field data. For convenience, these slabs are designated as "Taylor's Flat plate," "Heiman's flat slab", "Jokinen's flat slab", "Sbaronis flat plate", "Rangan's flat slab" and finally "Jawad flat slab". Table 3 shows the summary of available data of these slabs.

Table 3: Summary of slabs data

Structure	Type	Dimension (m)	Thickness (mm)	Sustained load (kN/m2)	Modulus of Elasticity(MPa)
Taylor and Heiman[24]	Flat plate	6.34×5.07	200	5.5	21400
Heiman[24]	Flat slab	7.54×7.24	240	5.5	28500
Jokinen and Scanlon[25]	Flat slab	9×9	200	5.5	27800
Sbarounis[26]	Flat plate	6.7×6.7	185	4.2	18000*
Rangan[27]	Flat slab	8.61×7.92	240	5.5	23200
Jawad[28]	Flat slab	7×7	220	7.3	21700

*Light weight concrete (1760 kg/m^3)

In this study, results of Monte Carlo simulation represented by probability of deflections (initial and long time) occurring, reinforced concrete floor system-optimum cost relationship, developed from sensitivity analysis, and results of comparative study between calculated and measured long-term deflections and slab thickness are presented. Simulation results are introduced in the form of PDF and CDF (probability density function and cumulative density function) respectively.

Figures (1) to (6) show the Monte Carlo simulation's results of immediate and time dependent deflection of reinforced concrete members (total probabilistic deflection of flat slab, flat plate, and two-way slab beam systems) obtained according to the approach presented in this work. Some selected figures are shown. A total of 54 figures (for the 27 slabs considered) are given in (Abdullah, 2008). For all shown figures, the probability density function is obtained by simulate deflection model of selected slab thickness of concrete slab. The horizontal axis shows the total deflections probability, and the vertical axis shows probability of these deflections. Then, by integrating the probability density function it is concluded that the cumulative density function (CDF) will be superimposed with (PDF) to obtain probability of failure. However, these results are preliminary results and they will be used to predict final and summary results, as explained in Figs. (7), (8), and (9).

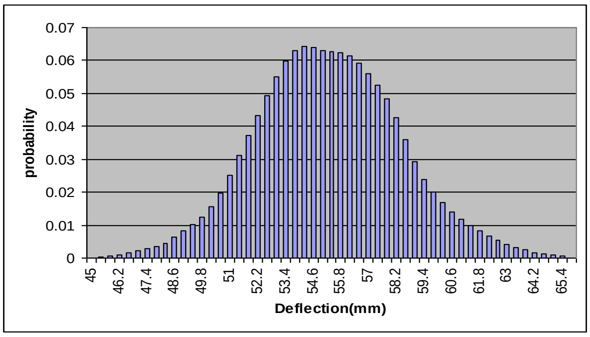

Figure 1: Probability histogram (PDF) for flat slab deflection Thickness (180mm)

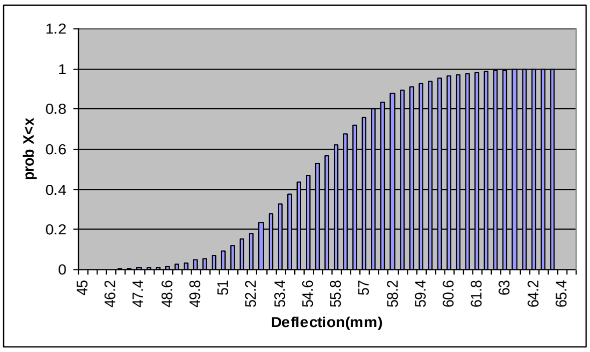

Figure 2: Cumulative histogram (CDF) for flat slab deflection Thickness (180mm)

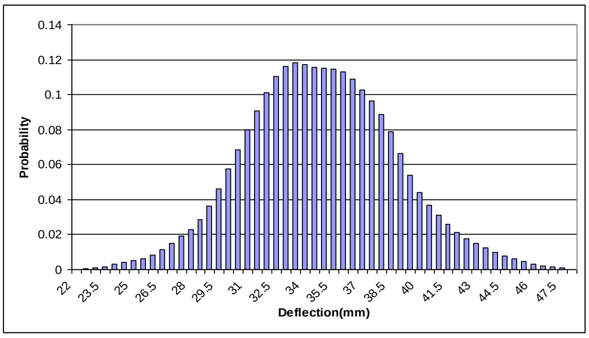

Figure 3: Probability histogram (PDF) for flat plate deflection Thickness (180mm)

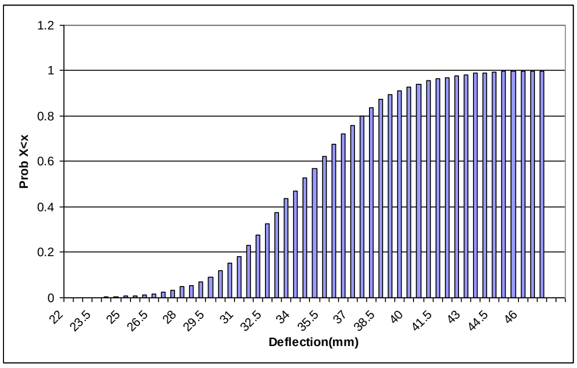

Figure 4: Cumulative histogram (CDF) for flat plate deflection Thickness (180mm)

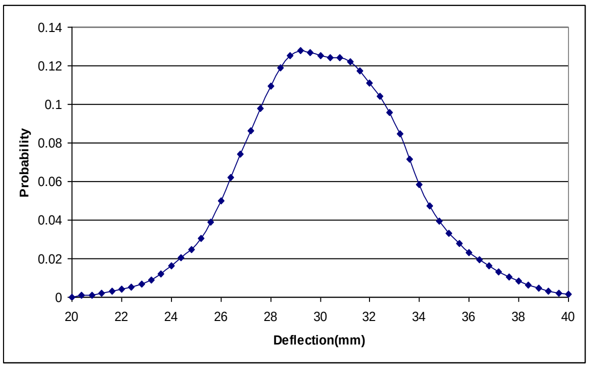

Figure 5: Probability distribution (PDF) for two-way slab beam deflection thickness (160mm)

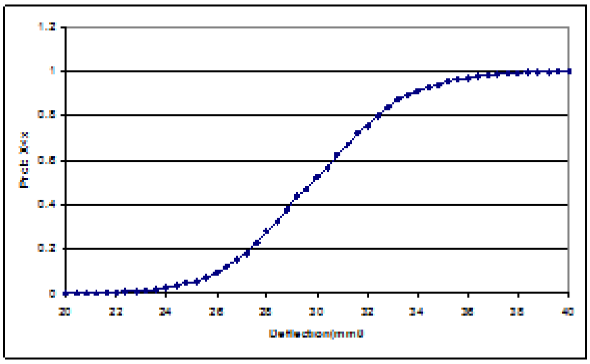

Figure (6): Cumulative distribution (CDF) for two-way slab beam deflection thickness (160mm)

Based on the results above, a slab-cost relationship is obtained by using the utility theory, as previously explained. The optimum cost in this study takes into account the initial construction cost plus probabilistic cost of failure (repair cost). Because cost data are highly dependent on local market conditions, construction costs and repair costs are taken as average present values with coefficient of variation (COV) equals to ±15% for both types of costs. Figures (7), (8), and (9) show the major results of this study represented by optimum thickness according to optimum cost by using the utility theory for three types of reinforced concrete floor systems (Flat plate, Flat slab, and two-way slab beam system).

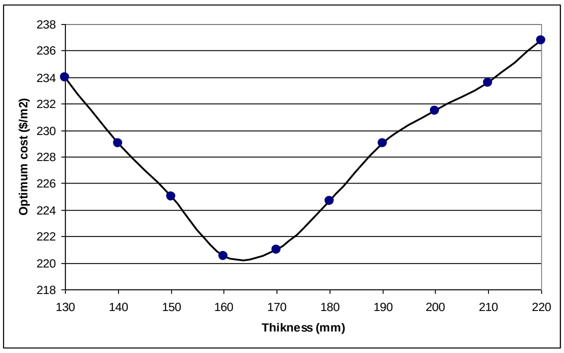

Figure 7: Optimum thickness of R-C flat plate system according to optimum cost

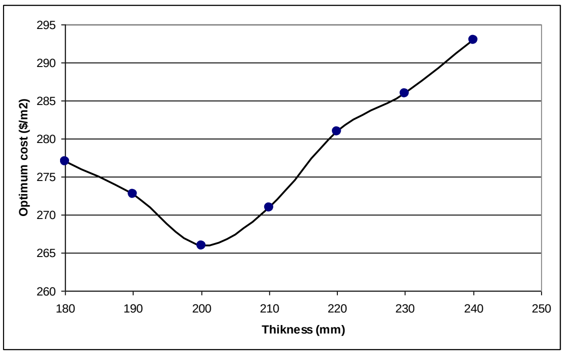

Figure 8: Optimum thickness of R-C flat slab system according to optimum cost

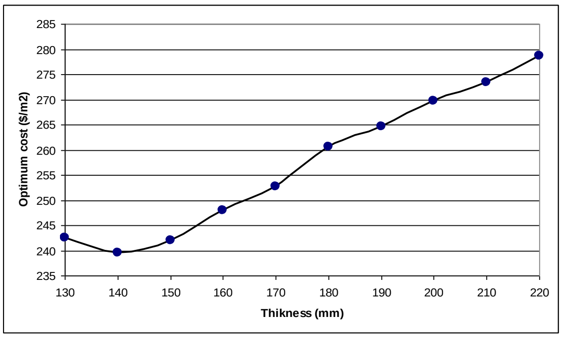

Figure 9: Optimum thickness of R-C slab-beam system according to optimum cost

From the three figures above, the optimum thicknesses for flat plate, flat slab, and slab-beam floors are (160) mm, (200) mm, and (140) mm respectively. The flat plate model for example, with optimum thickness equals to (160) mm, is lower than (180) mm when using ACI 318 Recommendations (1) (l_n/33). A question will introduce itself, why thickness of (160) mm is optimum thickness? The answer would be, as follows: slab with thicknesses 130, 140, and 150 mm are more economical but the probability of failure caused from excessive deflection to the allowable limits is higher than slab with thickness of 160 mm. On the other hand, slabs with 170, 180, 190, etc… are more expensive cost but there has a low probability of failure. Therefore, based on this analysis, one can say that the (160) mm slab thickness is an optimum slab thickness.

11. Sensitivity Analysis

A sensitivity analysis is developed to study the impact of changes in models input parameters on the solution, such as effect of variability of span length on (L/H) ratio according to optimum thickness obtained in previous section. Figures (10), (11), and (12) show variability of (L/H) with span length for flat plate, flat slab, and two-way beam slab.

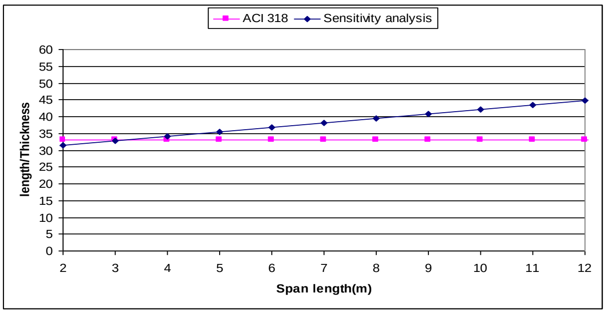

Figure 10: Effect of span length on (length / thickness) ratio for flat plates

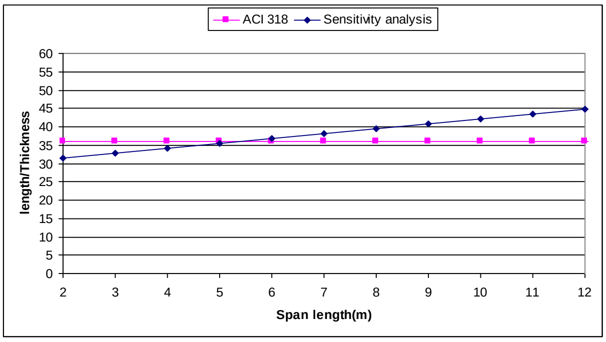

Figure 11: Effect of span length on (length / thickness) ratio for flat slab

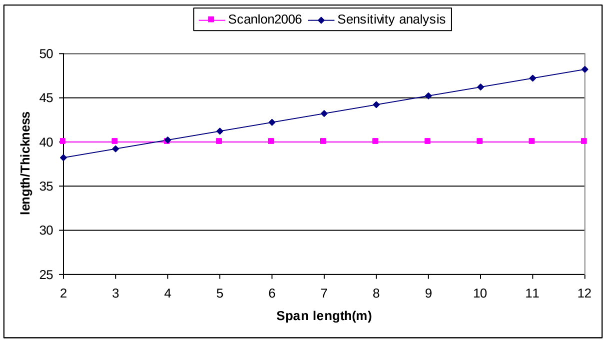

Figure 12: Effect of span length on (length / thickness) ratio for two-way beam slabs

12. Comparison between Thicknesses Obtained by Proposed Method and Current ACI 318-14 Code Minimum Thicknesses

A parametric study is performed to compare the thickness of two-way floor system obtained by proposed procedure and the minimum thickness values given in (ACI 318-14). The range of parameters considered and the results are presented, as shown in Figs. (10), (11) and (12). In each case, the ACI span-to-depth value is constant with span length, whereas the optimum span-to-depth ratio based on minimum total cost increases as span length increases. In general, the proposed procedure results in smaller thicknesses than ACI 318-14 for longer spans and larger thicknesses than ACI 318-14 for shorter spans. The results suggest that the ACI 318-14 minimum thickness rules are adequate for spans less than approximately (4 m) for Flat plate, (6 m) for Flat slab, and (4 m) for two-way beam system. The increases in span length will cause increases in total optimum cost which required to be applied in utility theory to estimate optimum thickness taking into account the uncertainties in member behavior and loads as well as lack of well-defined discrete serviceability.

13. Application and Comparative Study

Applications and comparative study are made between calculated and measured long-term deflections and slab thickness for interior panels of six multistory buildings reported by many authors such as (Taylor & Heiman, 1977; Jokinen & Scanlon, 1987; Sbarounis, 1984; Rangan, 1976; Jawad, 2000). These are fully documented field data. For convenience, these slabs are designated as "Taylor's flat plate", "Heiman's flat slab", "Jokinen's flat slab", "Sbaronis flat plate", "Rangan's flat slab", and finally "Jawad flat slab." Table 3 shows the summary of available data of these slabs, Table 4 compares the predictions of this study with the field-measured deflections and Table 5 compares the predictions of slab thickness obtained by utility theory with filed-measured dimensions.

Table 4: Comparison of measured and calculated deflections

Author / Reference	Measured Deflections (mm)	Calculated Deflections* (mm)	Calculated / Measured
Taylor and Heiman, 1977	24.4(9-year)	21.7	0.88
Heiman, 1977	23.6(9-year)	26.66	1.12
Jokinen and Scanlon, 1987	33(1-year)	39.57	1.19
Sbarounis, 1984	34.3(1-year)	30.6	0.89
Rangan, 1976	22.3(3.5year)	27.72	1.24
Jawad, 2000	33.6(20-year)	29.43	0.87

Mean= 1.033, COV=16%
*: sensitivity analysis

Table 5: Comparison of measured and calculated slab thickness

Author	Measured Thickness (mm)	Calculated Thickness* (mm)	Calculated / Measured
Taylor and Heiman, 1977	200	180	0.90
Heiman, 1977	240	200	0.83
Jokinen and Scanlon, 1987	200	220	1.10
Sbarounis, 1984	185	180	0.97
Rangan, 1976	240	220	0.91
Jawad, 2000	220	190	0.86

Mean= 0.929, COV=10% ,* sensitivity analysis

From the results of the tables above, the correlation between calculated and measured defections and slab thickness is shown to be good. The mean value of calculated/measured deflections is (1.033) with a coefficient of variation of 16 percent, and the mean value of calculated/measured slab thickness is (0.929) with a coefficient of variation of 10 percent. Figures (13) and (14) show a comparison of measured and calculated deflections and slab thickness for the six buildings.

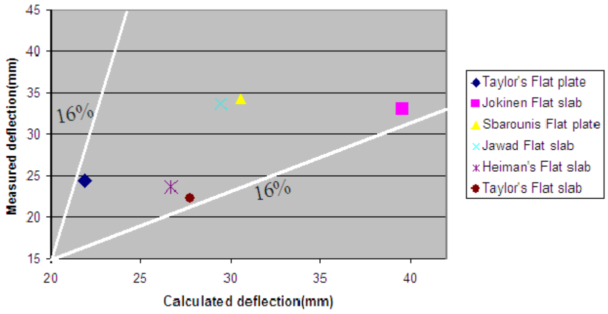

Figure 13: Comparison of measured and calculated deflections for the six buildings

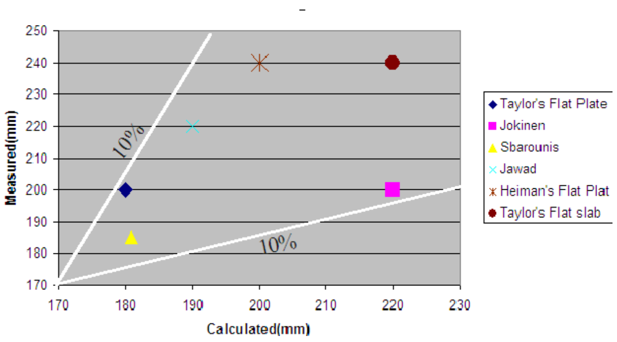

Figure 14: Comparison of measured and calculated slab thickness for the six buildings

14. Conclusion

The model presented in this study provides a rational approach to deflection control considering uncertainties in structural behavior and deflection limits and recognizes that the problem is subject to wide and unavoidable variability. The methodology has the potential to produce improvement in design codes related to serviceability. Based on utility theory and sensitivity analysis, the results suggest that the ACI 318-14 minimum thickness rules in all given span lengths are adequate for minimum thickness to approximately (4 m) span or less for flat plate floor system and that factors of span-depth ratio should be in the range between (33- 45) based on span length.

Based on utility theory and sensitivity analysis, the results suggest that the ACI 318-14 minimum thickness rules in all given span lengths are adequate for minimum thickness to approximately (6 m) span or less for flat slab floor system and that factors of span-depth ratio should be in the range between (36- 45) based on span length. Based on utility theory and sensitivity analysis, the results suggest that the ACI 318 minimum thickness rules in all given span lengths are adequate for minimum thickness to approximately (4 m) span or less for Two-way beam floor system and that factors of span-depth ratio should be in the range between (40- 48) based on span length. The method presented in this study has been applied to compute long- term deflections at midpoint of interior panels of floors of six existing buildings. A good agreement has been found to exist between measured and calculated values. The results of slab thicknesses summarized according to sensitivity analysis are compared with several field measured thicknesses and show good agreement with them.

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