Characterizations of $\tilde{S}p_c$-Open Sets and $\tilde{S}p_c$- almost Continuous Mapping in Soft Topological Spaces

Qumri H. Hamko[1] & Nehmat K. Ahmed[2]

[1&2]University of Salahaddin, Mathematics Department, College of Education, Erbil, Iraq
Correspondence : Qumri H. Hamko, University of Salahaddin, Erbil, Iraq.
Email: qumri.hamko@su.edu.krd

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Abstract: The purpose of this paper is to introduce the concepts of $\tilde{s}p_c$ – closure and $\tilde{s}p_c$ – interior operations and study the properties of $\tilde{s}p_c$ – closure and $\tilde{s}p_c$ – interior of a soft set in a soft topological space, also we aim to study properties of $\tilde{s}p_c$– derived, $\tilde{s}p_c$– frontier and $\tilde{s}p_c$– Exterior of soft set using the concept of $\tilde{s}p_c$– open set. Finally we introduce the concept of almost $\tilde{s}p_c$– continuous mapping and investigate properties and characterizations of these new types of mapping.

Keywords: Soft Closed Set, Soft Pre–Open Set, $\tilde{s}p_c$–Open Set And $\tilde{s}p_c$–Almost Continuity

1. Introduction

The concept of a soft set theory first introduced by the Russian researcher D. Molodtsov in 1999. He presented the fundamental results of the new theory and applied it successfully to several directions, such as game theory, Riemann integration, theory of measurement, probability theory and etc. A soft set is a collection of approximate descriptions of objects. Soft system provides general framework with the involvement of parameters. Hence in various fields, the researchers worked on soft set theory and its applications. Recently Shabir and Naz introduced the concept of soft topological space. Then same researchers have begun to study basic concepts and properties of soft topological spaces. Zorlutuna (2014) proved that a fuzzy topological space is a special case of soft topological spaces, and ordinary topological space can be considered a soft topological space In the present study, we introduce the concepts of $\tilde{s}p_c$ – closure and $\tilde{s}p_c$ – interior operations and study there properties in a soft topological space, also we objective and study properties of $\tilde{s}p_c$-derived, $\tilde{s}p_c$- frontier and $\tilde{s}p_c$-Exterior of soft set using the concept of $\tilde{s}p_c$-open set. Also we define almost $\tilde{s}p_c$-continuous mapping and investigate the properties and characterizations of these new types of mapping.

2. Preliminaries

Throughout this paper $\tilde{X}$ will always denote to soft topological spaces. If (F, A) is a subset of of the space$\tilde{X}$, $\tilde{s}p_c cl(F, A)$ and $\tilde{s}p_c int(F, A)$ denote the $soft\, p_c$ –closure and $soft\, p_c$ –interior of (F, A) respectively. A soft topological space is called soft locally indiscrete (Al-kadi, 2014), every soft open set over X is soft closed and $\tilde{X}$ is said to be extremally soft disconnected (Ahmed &

Hamko, 2018) if the soft closure of every soft open set is soft open.

Definition 2. 1. (llango & Ravindran, 2015) Let τ be a collection of soft sets over X. Then τ is said to be a soft topology on Xif:

1. $\widetilde{\emptyset}$, $\tilde{X}$ belong to τ .
2. the union of any number of soft sets in τ belongs to τ .
3. the intersection of any two soft sets in τ belongs to τ .

The triple $(\mathrm{X}, \tau, \mathrm{A})$ is called a soft topological space over X .

Let $(\mathrm{X}, \tau, \mathrm{A})$ be a soft space over $\tilde{X}$, then the members of τ are said to be soft open sets in $\tilde{X}$ denoted by $\mathrm{SO}(\mathrm{X})$and their complement are said to be soft closed sets in $\tilde{X}$, If Y is a non-empty subset of $\tilde{X}$, then $\tau_{\mathrm{Y}} = \{(\mathrm{F}, \mathrm{A}) \cap \mathrm{Y} : (\mathrm{F}, \mathrm{A}) \in \tau\}$is said to be the soft relative topology on Y and (Y, τ_Y, A) is said to be soft subspace of (X, τ, A).

Definition 2.2: (Ahmed & Hamko, 2018) A soft pre-open set (F, A) in a soft topological space $(\mathrm{X}, \tau, \mathrm{A})$ is called soft p_c-open if for each $x_\alpha \in (F, A)$, there exists a soft closed set (K, A) such that $x_\alpha \in (K, A) \subseteq (F, A)$. The family of all $\tilde{s}p_c$-open sets in a soft topological space $(\mathrm{X}, \tau, \mathrm{A})$ is denoted by $\tilde{s}\mathrm{pcO}(\mathrm{X}, \tau, \mathrm{A})$ or $\tilde{s}\mathrm{pcO}(\mathrm{X})$

Theorem 2.3: (Tozlu & Yksel, 2014) A soft topological space (X, τ, A) is soft regular if and only if for every $x \in X$ and every soft open set (F, A) containing x, there is a soft open set (G, A) of x such that $x \in (G, A) \subseteq \tilde{s}cl(G, A) \subseteq (F, A)$

Proposition 2.4: (Ahmed & Hamko, 2018) Let $\{(F_\lambda, A) : \lambda \in \Lambda\}$ be a collection of $\tilde{s}\mathrm{p_c}$ –open sets in a soft topological space , then $\cup \{(F_\lambda, A) : \lambda \in \Lambda\}$ is $\tilde{s}\mathrm{p}_c$-open

Proposition 2.5: (Akdag & Ozkan, 2014) Let (X, τ, A) be a soft topological space. If $(F, A) \in \tilde{s}\alpha O(X)$ and $(G, A) \in \tilde{s}PO(X)$, then $(F, A) \cap (G, A) \in \tilde{s}PO(X)$.

Corollary 2.6: (Ahmed & Hamko, 2018) For any soft subset (F, A) of a soft space $(\mathrm{X}, \tau, \mathrm{A})$. The following statements are equivalent:

1. (F, A) is soft clopen.
2. (F, A) is $\tilde{s}p_c$-open
3. (F, A) is soft preopen and soft closed

Theorem 2.7 (Akdag & Ozkan, 2014) Let $(\mathrm{F}, \mathrm{A}) \subseteq \tilde{\mathrm{Y}} \subseteq \tilde{\mathrm{X}}$, where $(\mathrm{X}, \tau, \mathrm{A})$ is a soft topological space and $\tilde{\mathrm{Y}}$ is a soft pre –open subspace of $\tilde{\mathrm{X}}$. $(\mathrm{F}, \mathrm{A}) \in \tilde{s}\mathrm{pO}(\mathrm{X})$, if and only if $(\mathrm{F}, \mathrm{A}) \in \tilde{s}\mathrm{pO}(\mathrm{Y})$

Theorem 2.8: (llango & Ravindran, 2016) If U is soft open and (F, A) is soft preopen, then $U \cap (F, A)$ is soft preopen.

Proposition 2.9. (Mussa & Khalaf, 2015) Let (Y, τ_Y, A) be a soft subspace of a soft space $(\mathrm{X}, \tau, \mathrm{A})$. If (F, A) is soft closed subset in $\tilde{X}$ and $(F, A) \subset \tilde{Y}$, then (F, A) is soft closed in $\tilde{\mathrm{Y}}$.

Proposition 2.10: (Mussa & Khalaf, 2015) Let (X, τ, A) be a soft topological space. If $(F, A) \in \tilde{s}\alpha O(X)$ and $(G, A) \in \tilde{s}PO(X)$, then $(F, A) \cap (G, A) \in \tilde{s}P\,O(X)$.

Dentition 2.11. (Al-kadi, 2014) Let $SS(X)_E$ and $SS(Y)_K$ be soft classes. Let $u : X \to Y$ and $p : E \to K$ be mappings. Then a soft mapping $f_{pu}: SS(X)E \to SS(Y)K$ is define as:
(1) For a soft set (F, A) in $SS(X)_E$, $(f_{pu}(F, A), B), B = p(A) \subseteq K$ is a soft set in $SS(Y)_K$ given by

$$f_{pu}(F, A)(\beta) = \begin{cases} u\left(\cup_{\alpha \in p^{-1}(\beta) \cap A} \left(F(\alpha)\right)\right) & , p^{-1}(\beta) \cap A \neq \emptyset \\ \emptyset & otherwise \end{cases}$$

for $\beta \in B \subseteq K$. $(f_{pu}((F, A), B)$ is called a soft image of a soft set (F, A). If $B = K$, then we shall write $(f_{pu}(F, A), K)$ as $f_{pu}(F, A)$.
(2) For a soft set (G, C) in $SS(Y)_K$. $(f_{pu}^{-1}(G, C), D)$, $D = p^{-1}(C)$ is a soft set in $SS(X)_E$ given by

$$f_{pu}^{-1}(G, C)(\alpha) = \begin{cases} u^{-1}\left(G\left(p(\alpha)\right)\right) & p(\alpha) \in C \\ \emptyset & otherwise \end{cases}$$

for $\alpha \in D \subseteq E$,$(f_{pu}^{-1}(G, C), E)$ is called a soft inverse image of a soft set (G, C).
We shall write $(f_{pu}^{-1}(G, C), E)$ as $f_{pu}^{-1}(G, C)$.

Proposition 2.12: (Ahmed & Hamko, 2018) If a space $\tilde{X}$ is a soft T_1-space, then $\tilde{s}p_c OP(X) = \tilde{s}PO(X)$

Proposition 2.13: (Ahmed & Hamko, 2018) Let (Y, τ_Y, A) be a subspace of a space (X, τ, A) and $(F, A) \subseteq \tilde{Y}$. If (F, A) is $\tilde{s}p_c$ -open in a subspace (Y, τ_Y, A) and Y is soft clopen, then (F, A) is $\tilde{s}p_c$ -open set in $\tilde{X}$.

Lemma2.14. (llango & Ravindran, 2015) Let (F, A) be a soft subset of a soft space (X, τ, A). Then $(F, A) \in \tilde{s}P\,O(X)$,if and only if $\tilde{s}\tilde{s}cl(F, A) = \tilde{s}int\tilde{s}cl(F, A)$.

3. Operators on $\tilde{s}p_c$-Open Sets

In this section, we define and study some operators on soft topological spaces via the concept of $\tilde{s}p_c$-open sets such as $\tilde{s}p_c$-neighbourhood, $\tilde{s}p_c$-derived, $\tilde{s}p_c$-interior, $\tilde{s}p_c$-closure and$\tilde{s}p_c$-boundary.

Definition 3.1: A soft set (F, A) in a soft topological space (X, τ, A) is called $\tilde{s}p_c$-neighbourhood of a soft point $x_\alpha \in SP(X)_A$ if there exists an $\tilde{s}p_c$ -open set (G, A) such that $x_\alpha \in (G, A) \subseteq (F, A)$.

Proposition 3.2: A soft set (F, A) over a soft space $\tilde{X}$ is $\tilde{s}p_c$-open if and only if (F, A) is an $\tilde{s}p_c$-neighbourhood of each of its soft points.

Proof. Let (F, A) be $\tilde{s}p$ -open and $x_\alpha \in (F, A)$. Then $x_\alpha \in (F, A) \subseteq (F, A)$. Therefore, (F, A) is an $\tilde{s}p_c$-neighbourhood of x_α. Conversely, let (F, A) be an $\tilde{s}p_c$-neighbourhood of each of its soft points. Let $x_\alpha \in (F, A)$. Since (F, A) is an $\tilde{s}p_c$-neighbourhood of each of its soft points, there exists an $\tilde{s}p_c$-open set (G, A) such that $x_\alpha \in (G, A) \subseteq (F, A)$. Therefore, $(F, A) = \cup \{x_\alpha\} \subseteq \cup (G, A) \subseteq (F, A)$

for each $x_\alpha \in (F, A)$. It follows that (F, A) is a union of $\tilde{s}p_c$-open sets and hence (F, A) is $\tilde{s}p_c$-open.

Proposition 3.3: For any two soft subsets (F, A), (G, A) of a soft topological space $\tilde{X}$ with $(F, A) \subseteq (G, A)$, if (F, A) is an $\tilde{s}p_c$-neighbourhood of a soft point $x_\alpha \in SP(X)_A$, then (G, A) is also a $\tilde{s}p_c$-neighbourhood of the same soft point.

Proof: Straightforward.

Remark 3.4: Every $\tilde{s}p_c$-neighbourhood of a soft point is soft pre-neighbourhood.

Definition 3.5: Let (X, τ, A) be a soft topological space. A soft point $x_\alpha \in SP(X)_A$ is said to be an $\tilde{s}p_c$ -limit soft point of a soft set (F, A) if for every $\tilde{s}p_c$-open set (G, A) containing x_α, then $(G, A) \cap [(F, A) \backslash \{x_\alpha\}] \neq \tilde{\emptyset}$. The set of all $\tilde{s}p_c$-limit soft points of (F, A) is called an $\tilde{s}p_c$-derived set of (F, A) and is denoted by $\tilde{s}p_c - D(F, A)$

Proposition 3.6:A soft set (F, A) of a soft topological space (X, τ, A) is $\tilde{s}p_c$ -closed if and only if (F, A) contains all its $\tilde{s}p_c$-limit soft points.

Proof. Let (F, A) be $\tilde{s}p_c$-closed and $x_\alpha \notin (F, A)$, then $X \setminus (F, A)$ is $\tilde{s}p_c$-open and $x_\alpha \in X \setminus (F, A)$. Since $(F, A) \cap [X \setminus (F, A)] = \tilde{\emptyset}$, x_α cannot be an $\tilde{s}p_c$-limit soft point of (F, A). Therefore, (F, A) contains all its $\tilde{s}p_c$-limit soft points. Conversely, let (F, A) contains all its $\tilde{s}p_c$-limit soft points. Let $x_\alpha \in X \setminus (F, A)$. By our assumption, $x_\alpha$is not an $\tilde{s}p_c$-limit soft point of (F, A). Then there exists an $\tilde{s}p_c$ -open set (G, A) such that $x_\alpha \in (G, A)$ *and* $(F, A) \cap (G, A) = \tilde{\emptyset}$. Therefore, $(G, A) \subseteq X \setminus (F, A)$. So $X \setminus (F, A) = \cup \{x_\alpha\} \subseteq \cup (G, A) \subseteq X \setminus (F, A)$ for each$x_\alpha \in X \setminus (F, A)$. Thus $X \setminus (F, A)$ is $\tilde{s}p_c$-open and hence (F, A) is $\tilde{s}p_c$-closed.

Proposition 3.7: Let (F, A) be a soft subset over $\tilde{X}$. If for each soft closed set (H, A) of $\tilde{X}$ containing x_α such that $(H, A) \cap [(F, A) \backslash \{x_\alpha\}] \neq \tilde{\emptyset}$, then a soft point $x_\alpha \in SP(X)_A$ is an $\tilde{s}p_c$-limit soft point of (F, A).

Proof: Let (G, A) be any $\tilde{s}p_c$-open set containing x_α, then for each $x_\alpha \in (G, A) \in \tilde{s}pO(X)$, there exists a soft closed set (H, A) such that $x_\alpha \in (H, A) \subseteq (G, A)$. By hypothesis, we have $(H, A) \cap [(F, A) \backslash \{x_\alpha\}] \neq \tilde{\emptyset}$ Hence $(G, A) \cap [(F, A) \backslash \{x\alpha\}] \neq \tilde{\emptyset}$ Therefore, a soft point $x_\alpha \in SP(X)_A$ is an $\tilde{s}p_c$-limit soft point of (F, A).

Proposition 3.8: For soft subsets (F, A) and (G, A) over $\tilde{X}$, the following statements are true:

1. $\tilde{s}p_c D(\tilde{\emptyset}) = \tilde{\emptyset}$.
2. If $x_\alpha \in \tilde{s}p_c D(F, A)$, then $x_\alpha \in \tilde{s}p_c D((F, A) \setminus \{x_\alpha\})$.
3. If $(F, A) \subseteq (G, A)$, then $\tilde{s}p_c D(F, A) \subseteq \tilde{s}p_c D(G, A)$
4. $\tilde{s}p_c D(F, A) \cup \tilde{s}p_c D(G, A) \subseteq \tilde{s}p_c D((F, A) \cup (G, A))$
5. $\tilde{s}p_c D(F, A) \cap \tilde{s}p_c D(G, A) \supseteq \tilde{s}p_c D((F, A) \cap (G, A))$

Proof: Follows directly from definition 3.5 and Proposition 3.6.

In general, $\tilde{s}p_cD(F,A) \cup \tilde{s}p_cD(G,A) \neq \tilde{s}p_cD((F,A) \cup (G,A))$ and $\tilde{s}p_cD(F,A) \cap \tilde{s}p_cD(G,A) \neq \tilde{s}p_cD((F,A) \cap (G,A))$, as it is shown in the following two examples:

Example 3.9: Consider $X = \{a, b\}$, $A = \{u, s\}$ $\tau = \{\widetilde{\emptyset}, \tilde{X}, (F_1, A), (F_2, A)(F_3, A), (F_4, A), (F_5, A), (F_6, A)\}$ Where $F_1(u) = \{a\}$ $F_1(s) = \widetilde{\emptyset}$ $F_2(u) = \{a\}$ $F_2(s) = \{b\}$ $F_3(u) = \{b\}$ $F_3(s) = \{a\}$

$F_4(u) = \{\{a, b\}$ $F_4(s) = \{a\}$ $F_5(u) = \{b\}$ $F_5(s) = \{a, b\}$

$F_6(u) = \widetilde{\emptyset}$ $F_6(s) = \{b\}$,

Then$\tilde{s}p_cO(X) = \tau$ if we define $(F,A), (G,A)$ and $(H,A) = (F,A) \cup (G,A)$ by $F(u) = \{b\}, F(s) = \{a,b\}$, $G(u) = \{a\}, G(s) = \{a\}$ $H(u) = \{a,b\}, H(s) = \{a,b\}$. Then $\tilde{s}p_cD(F,A) = \{b_u, a_s\}$, $\tilde{s}p_cD(G,A) = \{b_u\}$ and $\tilde{s}p_cD((H,A) = SP(X)_A$, . It follows that $\tilde{s}p_cD(F,A) \cup \tilde{s}p_cD(G,A) \neq \tilde{s}p_cD((F,A) \cup (G,A))$.

Example 3.10: Let R be the set of all real numbers, $A = \{u, s\}$, and $\beta = \{(F_a^b, A); a < b\}$ where the map $F_a^b: A \to P(R)$ defined as follows:

$$F_a^b(\alpha) = \begin{cases} (a,b) \ if \ \alpha = u \\ R \ or \ \widetilde{\emptyset} \ if \ \alpha = s \end{cases}$$

Let τ be the topology on R with the base β. Then the set

$G_a^b(\alpha) = \begin{cases} Q \ \ if \ \alpha = u \\ \widetilde{\emptyset} \ \ if \ \alpha = s \end{cases}$ where Q is the set of all rational numbers. If we take

$$G_1(\alpha) = \begin{cases} (0,1) & if \ \alpha = u \\ \widetilde{\emptyset} & if \ \alpha = s \end{cases} \qquad G_2(\alpha) = \begin{cases} (1,2) & if \ \alpha = u \\ \widetilde{\emptyset} & if \ \alpha = s \end{cases}$$

$G_3(\alpha) = G_1(\alpha) \cap G_2(\alpha)$ so that $G_3(\alpha) = \begin{cases} \widetilde{\emptyset} & if \ \alpha = u \\ \widetilde{\emptyset} & if \ \alpha = s \end{cases}$ Then $\tilde{s}p_cDG_1(\alpha) = [0,1]$, $\tilde{s}p_cDG_2(\alpha) = [1,2]$, hence $\tilde{s}p_cDG_1(\alpha) \cap \tilde{s}p_cDG_2(\alpha) = \{1\}$, but $\tilde{s}p_cDG_3(\alpha) = \widetilde{\emptyset}$. It follows that $\tilde{s}p_cD(F,A) \cap \tilde{s}p_cD(G,A) \neq \tilde{s}p_cD((F,A) \cap (G,A))$

Proposition 3.11: If (F,A) and (G,A) be soft subsets over $\tilde{X}$, then we have the following properties:

1. $\tilde{s}p_c\, D(\tilde{s}p_c\, D(F,A)) \setminus (F,A) \subseteq \tilde{s}p_c\, D(F,A)$.
2. $\tilde{s}p_cD((F,A) \cup \tilde{s}p_c\, D(F,A)) \subseteq (F,A) \cup \tilde{s}p_c\, D(F,A)$.

Proof: (1) If $x_\alpha \in \tilde{s}p_c\, D(\tilde{s}p_c\, D(F,A)) \setminus (F,A)$ implies that $x_\alpha \in \tilde{s}p_c\, D(\tilde{s}p_c\, D(F,A))$ and (G,A) is a $\tilde{s}p_c$ −open set containing x_α, then $(G,A) \cap [\tilde{s}p_c\, D(F,A) \setminus \{x\alpha\}] \neq \widetilde{\emptyset}$. Let $y_\beta \in (G,A) \cap [\tilde{s}p_c\, D(F,A) \setminus \{x_\alpha\}]$. Then , since $y_\beta \in \tilde{s}p_c\, D(F,A)$ and $y_\beta \in (G,A), (G,A) \cap [(F,A) \setminus \{y\beta\}] \neq \widetilde{\emptyset}$. Let $z_\gamma \in (G,A) \cap [(F,A) \setminus \{y_\beta\}]$. Then $z_\gamma \neq x_\alpha$ for $z_\gamma \in (F,A)$ and $x_\alpha \notin (F,A)$.

Hence, $(G,A) \cap [(F,A) \setminus \{x_\alpha\}] \neq \tilde{\emptyset}$. Therefore, $x_\alpha \in \tilde{s}p_c D(F,A)$.

(2) Let $x_\alpha \in \tilde{s}p_c D((F,A) \cup \tilde{s}p_c D(F,A))$. If $x_\alpha \in (F,A)$, the result is obvious. So, let $x_\alpha \in \tilde{s}p_c D((F,A) \cup \tilde{s}p_c D(F,A)) \setminus (F,A)$, then for any $\tilde{s}p_c$ −open set (G,A) containing x_α we have $(G,A) \cap ([(F,A) \cup \tilde{s}p_c D(F,A)] \setminus \{x_\alpha\}) \neq \tilde{\emptyset}$. Thus $(G,A) \cap [(F,A) \setminus \{x_\alpha\}] \neq \tilde{\emptyset}$ or $(G,A) \cap [\tilde{s}p_c D(F,A) \setminus \{x_\alpha\}] \neq \tilde{\emptyset}$. Now it follows similarly from (1) that $(G,A) \cap [(F,A) \setminus \{x_\alpha\}] \neq \tilde{\emptyset}$. Hence, $x_\alpha \in \tilde{s}p_c D(F,A)$. Therefore, in both cases, we get $\tilde{s}p_c D((F,A) \cup \tilde{s}p_c D(F,A)) \subseteq (F,A) \cup \tilde{s}p_c D(F,A)$.

Proposition 3.12: For a soft subset (F,A) over a soft space $\tilde{X}$, $\tilde{s}pD(F,A) \subseteq \tilde{s}p_c D(F,A)$.

Proof. Follows from the fact that every $\tilde{s}p_c$ -open set is soft pre-open.

The following example shows that $\tilde{s}p_c D(F,A) \not\subseteq \tilde{s}pD(F,A)$.

Example 3.13: Consider $U = \{u_1, u_2, u_3\}$, $E = \{e_1, e_2, e_3\}$, A = $\{e_1, e_2\}$, and $F_A = \{(e_1,\{u_1\}),(e_2,\{u_1,u_2\})\}$. The class of all soft subsets over U is denoted by $S(F_A)$. Then $F_{A_1} = \{(e_1,\{u_1\})\}$, $F_{A_2} = \{(e_1,\{u_1\}),(e_2,\{u_1\})\}$, $F_{A_3} = \{(e_1,\{u_1\}),(e_2,\{u_2\})\}$, $F_{A_4} = \{(e_2,\{u_1,u_2\})\}$, $F_{A_5} = \{(e_2,\{u_1\})\}$, $F_{A_6} = \{(e_2,\{u_2\})\}$ $F_{A_7} = F_A$ $F_{A_8} = F_\emptyset$. Define the soft topology $\tau = \{F_A, F_\emptyset, F_{A_1}, F_{A_3}, F_{A_4}, F_{A_6}\}$, $\tilde{s}PO(X) = \{F_A, F_\emptyset, F_{A_1}, F_{A_3}, F_{A_4}, F_{A_6}\}$,

$\tilde{s}P_cO(X) = \{F_A, F_\emptyset, F_{A_1}, F_{A_4}\}$. if we take (F_6, A). Then $\tilde{s}pD(F_6,A) = \tilde{\emptyset}$, and $\tilde{s}p_c D(F_6,A) = \{u_{1_{e_2}}\}$, so that .$\tilde{s}p_c D(F,A) \not\subseteq \tilde{s}pD(F,A)$.

Definition 3.14: Let (F,A) be a soft subset of a soft topological space (X,τ,A). A soft point $x_\alpha \in SP(X)_A$ is said to be $\tilde{s}p_c$ -interior soft point of (F,A) if there exists an $\tilde{s}p_c$ -open set (G,A) such that $x_\alpha \in (G,A) \subseteq (F,A)$. The set of all $\tilde{s}p_c$-interior soft points of (F,A) is called $\tilde{s}p_c$-interior of (F,A) and is denoted by $\tilde{s}p_c int(F,A)$.

Proposition 3.15: Let (F,A) be a soft subset over $\tilde{X}$. If a soft point x_α is in the $\tilde{s}p_c$ -interior of (F,A), then there exists a soft closed set (H,A) of $\tilde{X}$ containing x_α such that $(H,A) \subseteq (F,A)$.

Proof. Suppose that $x_\alpha \in \tilde{s}p_c int(F,A)$, then there exists an $\tilde{s}p_c$-open set (G,A) containing x_α such that $(G,A) \subseteq (F,A)$. Since (G,A) is $\tilde{s}p_c$-open set, there exists a soft closed set (H, A) of $\tilde{X}$ containing x_α such that $(H,A) \subseteq (G,A) \subseteq (F,A)$. Hence $x_\alpha \in (H,A) \subseteq (F,A)$.

Corollary 3.16: Let $\tilde{X}$ be a soft topological space and (F,A) be any soft set over $\tilde{X}$ and $x_\alpha \in SP(X)_A$. Then $x_\alpha$is an $\tilde{s}p_c$-interior soft point of (F,A) if and only if (F,A) is an $\tilde{s}p_c$-neighbourhood of x_α.

Proof. Obvious.

Some properties of $\tilde{s}p_c$ -interior soft sets are stated in the following Proposition and their proof are straightforward.

Proposition 2.17: Let (X, τ, A) be a soft topological space and (F, A) be any soft set over $\tilde{X}$, then

1. The $\tilde{s}p_c$-interior of (F, A) is the union of all $\tilde{s}p_c$-open sets which are contained in (F, A).
2. $\tilde{s}p_c$int(F, A) is $\tilde{s}p_c$-open set in $\tilde{X}$ contained in (F, A).
3. $\tilde{s}p_c int(F, A)$ is the largest $\tilde{s}p_c$-open set contained in (F, A).
4. (F, A) is $\tilde{s}p_c$ -open if and only if $(F, A) = \tilde{s}p_c int(F, A)$.
5. $\tilde{s}p_c int(\tilde{s}p_c int(F, A)) = \tilde{s}p_c int(F, A)$.

Some other properties of $\tilde{s}p_c$-interior of a soft set (F, A) are mentioned in the following statements:

Proposition 3.18. Let (X, τ, A) be a soft topological space and $(F, A), (G, A)$ be any soft sets over $\tilde{X}$, then

1. 1-$\tilde{s}p_c int(\tilde{\emptyset}) = \tilde{\emptyset}$ and $\tilde{s}p_c int(\tilde{X}) = \tilde{X}$.
2. 2- $\tilde{s}p_c int(F, A) \subseteq (F, A)$.
3. $3- If\ (F, A) \subseteq (G, A), then\ \tilde{s}p_c int(F, A) \subseteq \tilde{s}p_c int(G, A)$.
4. 4- $\tilde{s}p_c int(F, A) \cup \tilde{s}p_c int(G, A) \subseteq \tilde{s}p_c int((F, A) \cup (G, A))$.
5. 5- $\tilde{s}p_c int(F, A) \cap \tilde{s}p_c int(G, A) \supseteq \tilde{s}p_c int((F, A) \cap (G, A))$.

Proof Obvious.

In general, the equality of (4) and (5) do not hold as is illustrated in the following two examples:

Example 3.19: Consider $X = \{a, b\}$, and $E = \{0,1\}E = \{0,1\}$. We consider

$\tilde{s}p_c O(X) = \{\tilde{\emptyset}, \tilde{X}, (H_i, A); i = 1,2, \dots ,6\}$, where

$$H_1(x) = \begin{cases} \{a\} & if\ x = 0 \\ \emptyset & if\ x = 1 \end{cases} \quad H_2(x) = \begin{cases} \{b\} & if\ x = 0 \\ \{a\} & if\ x = 1 \end{cases} \quad H_3(x) = \begin{cases} \emptyset & if\ x = 0 \\ \{b\} & if\ x = 1 \end{cases}$$

$$H_4(x) = \begin{cases} \{a, b\} & if\ x = 0 \\ \{a\} & if\ x = 1 \end{cases} \quad H_5(x) = \begin{cases} \{a\} & if\ x = 0 \\ \{b\} & if\ x = 1 \end{cases} \quad H_6(x) = \begin{cases} \{b\} & if\ x = 0 \\ \{a, b\} & if\ x = 1 \end{cases}$$

If (F, A), (G, A) and $(H, A) = (F, A) \cup (G, A)$ define as $F(x) = \begin{cases} \emptyset & if\ x = 0 \\ \{a\} & if\ x = 1 \end{cases}$ $G(x) = \begin{cases} \{b\} & if\ x = 0 \\ \emptyset & if\ x = 1 \end{cases}$, $H(x) = \begin{cases} \{b\} & if\ x = 0 \\ \{a\} & if\ x = 1 \end{cases}$ $\tilde{s}p_c int(F, A) = \emptyset$ and $\tilde{s}p_c int(G, A) = \emptyset$, then $\tilde{s}p_c int(F, A) \cup \tilde{s}p_c int(G, A) = \emptyset$ and $\tilde{s}p_c int(H, A) = (H, A)$ It follows that $\tilde{s}p_c int(F, A) \cup \tilde{s}p_c int(G, A) \neq \tilde{s}p_c int((F, A) \cup (G, A))$.

Example 3.20 : Consider the co-finite soft topological space $\tilde{X}$, with the same set of parameters A, where $X = A = N$ the set of all natural numbers, $\tau = \{\tilde{\emptyset}, \tilde{X}\} \cup \{G_1(n), G_2(n)\}$, where $G: A \to P(X)$ such that $G_1(n) = \{ 2n, for\ every\ n \in N\}$ and $G_2(n) = \{ 2n + 1, for\ every\ n \in N\} \cup \{0\}$, easily can be checked that both $G_1(n)$, $G_2(n)$ are soft pre open sets and since co-finite soft topology is soft T_1-space then by Proposition 2.12 $G_1(n)$, $G_2(n)$ are $\tilde{s}p_c$ -open sets, therefore $\tilde{s}p_c intG_1(n) \cap \tilde{s}p_c intG_2(n) = G_1(n) \cap G_2(n) = \{0\}$ but $\tilde{s}p_c int(G_1(n) \cap G_2(n) = \tilde{s}p_c int\{0\} = \tilde{\emptyset}$. It follows that $\tilde{s}p_c int(F,A) \cap \tilde{s}p_c int(G,A) \neq \tilde{s}p_c int((F,A) \cap (G,A))$.

Proposition 3.21. For a soft subset (F,A) over $\tilde{X}$, we have $\tilde{s}p_c int(F,A) \subseteq \tilde{s}pint\ (F,A)$

Proof . Follows from the fact that every $\tilde{s}p_c$-open set is soft pre-open.

The following example shows that the converse of Proposition 3.21 does not hold in general.

Example 3.22:Consider $U = \{u_1, u_2, u_3\}$, $A = \{e_1, e_1\}$ and $(F,A) = \{(e_1, \{u_1\}), (e_2, \{u_1, u_2\})\}$. The class of all soft subsets over U is denoted by $SF(A)$. Then $(F_1, A) = \{(e_1, \{u_1\})\}$, $(F_2, A) = \{(e_1, \{u_1\}), (e_2, \{u_1\})\}$, $(F_3, A) = \{(e_1, \{u_1\}), (e_2, \{u_2\})\}$, $(F_4, A) = \{(e_2, \{u_1, u_2\})\}$, $(F_5, A) = \{(e_2, \{u_1\})\}$, $(F_6, A) = \{(e_2, \{u_2\})\}$,$(F_7, A) = (F, A)$, $(F_8, A) = (F, \emptyset)$. Define the soft topology $= \{(F_8, A), (F_7, A), (F_1, A), (F_3, A), (F_4, A), (F_6, A)\} = \tilde{s}po(X)$, and $\tilde{s}p_c o(X) = \{(F_8, A), (F_7, A), (F_1, A), , (F_4, A)\}$. If we take (F_3, A) then $\tilde{s}p_c int(F_3, A) = (F_1, A)$ and $\tilde{s}pint(F_3, A) = (F_3, A)$. Thus shows that $\tilde{s}p_c int(F,A) \not\supseteq \tilde{s}pint\ (F,A)$

Proposition 3.23: Let (F,A) be a soft subset of $\tilde{X}$. Then, $\tilde{s}p_c int(F,A) = (F,A) \backslash \tilde{s}p_c\ D(X \backslash (F,A))$.

Proof. If $x_\alpha \in (F,A) \backslash \tilde{s}p_c\ D(X \backslash (F,A))$, then $x_\alpha \notin \tilde{s}p_c\ D(X \backslash (F,A))$ and then, there exists an $\tilde{s}p_c$ -open set (G,A) containing x_α such that $(G,A) \cap X \backslash (F,A) = \tilde{\emptyset}$. Then, $x_\alpha \in (G,A) \subseteq (F,A)$ and hence $x_\alpha \in \tilde{s}p_c int(F,A)$, that is $(F,A) \backslash \tilde{s}p_c\ D(X \backslash (F,A)) \subseteq \tilde{s}p_c int(F,A)$. On the other hand, if $x_\alpha \in \tilde{s}p_c int(F,A)$, then $x_\alpha \notin \tilde{s}p_c D(X \backslash (F,A))$ since, $\tilde{s}p_c int(F,A)$ is $\tilde{s}p_c$ -open and $\tilde{s}p_c int(F,A) \cap (X \backslash (F,A)) = \tilde{\tilde{\emptyset}}$. Hence, $\tilde{s}p_c int(F,A) = (F,A) \backslash \tilde{s}p_c D(X \backslash (F,A))$.

Definition 3.24. Let (X, τ, A) be a soft topological space and (F,A) be a soft set over $\tilde{X}$. Then, $\tilde{s}p_c$-closure of (F,A) is denoted by $\tilde{s}p_c cl(F,A)$ and is defined as the intersection of all $\tilde{s}p_c$-closed super sets of (F,A).

Proposition 3.25. Let (X, τ, A) be a soft topological space and (F,A) be a soft set over $\tilde{X}$. Therefore a soft point $x_\alpha \in SP\ (X\ )_A$, the following are equivalent:

1. For any $\tilde{s}p_c$-open set (G,A) containing x_α,we have $(G,A) \cap (F,A) \neq \tilde{\tilde{\emptyset}}$.
2. $x_\alpha \in \tilde{s}p_c cl(F,A)$

Proof.$1 \rightarrow 2$: $x_\alpha \in \tilde{s}p_c\ cl(F,A)$, then there exists a $\tilde{s}p_c$-closed set (K,A) such that $(F,A) \subseteq (K,A)$ and $x_\alpha \notin \tilde{s}p_c cl(K,A)$. But $\backslash (K,A)$ $\tilde{s}p_c$-open set containing x_α And therefore $(F,A) \cap \tilde{X} \backslash (K,A) \subseteq (F,A) \cap \tilde{X} \backslash (F,A) = \emptyset$, which is contradiction. Hence $x_\alpha \in \tilde{s}p_c cl(F,A)$.

$2 \rightarrow 1$: Suppose there exists a $\tilde{s}p_c$ -open set (G, A) containingx_α such that $(G,A) \cap (F,A) = \tilde{\tilde{\emptyset}}$, then $(F,A) \subseteq \tilde{X} \backslash (G,A)$. Since $\tilde{X} \backslash (G,A)$ is $\tilde{s}p_c$ -closed set, $\tilde{s}p_c\ cl(F,A) \subseteq \tilde{\tilde{X}} \backslash (G,A)$. Hence $x_\alpha \notin \tilde{s}p_c\ cl(F,A)$ contradiction .

Corollary 3.26. Let (F, A) be any soft subset over $\tilde{X}$. If $(F, A) \cap (H, A) = \tilde{\emptyset}$, for every soft closed set (H, A) containing x_α, then the soft point $x_\alpha \in \tilde{s}pccl(F, A)$.

Proof. Suppose that (G, A) is any $\tilde{s}p_c$-open set containing x_α, then there exists a soft closed set (H, A) such that $x_\alpha \in (H, A) \subseteq (G, A)$. So by hypothesis $(F, A) \cap (H, A) = \tilde{\emptyset}$ which implies that $(F, A) \cap (G, A) = \tilde{\emptyset}$ for every $\tilde{s}p_c$-open set (G, A) containing x_α. Therefore, by Proposition 3.25, $x_\alpha \in \tilde{s}p_c cl(F, A)$.

Proposition 3.27. Let (F, A) be a soft subsets over $\tilde{X}$, *then* $\tilde{s}pccl(F, A) = (F, A) \cup \tilde{s}pcD(F, A)$.

Proof. Since $\tilde{s}p_c D(F, A) \subseteq \tilde{s}p_c\, cl(F, A)$ and $(F, A) \subseteq \tilde{s}p_c\, cl(F, A)$, then $(F, A) \cup \tilde{s}p_c\, D(F, A) \subseteq \tilde{s}p_c\, cl(F, A)$. On the other hand, Since $\tilde{s}p_c cl(F, A)$ is the smallest $\tilde{s}p_c$ -closed set containing (F, A), so it is enough to prove that $(F, A) \cup \tilde{s}p_c D(F, A)$ is $\tilde{s}p_c$-closed. Let $x_\alpha \notin ((F, A) \cup \tilde{s}p_c D(F, A))$. This implies that $x_\alpha \notin (F, A)$ *and* $x_\alpha \notin \tilde{s}p_c\, D(F, A)$, which mean that there exists an $\tilde{s}p_c$ -open set (G, A) of x_α which contains no soft point of (F, A)other than x_α and $x_\alpha \notin (F, A)$. So (G, A) contains no soft point of (F, A), which implies that $(G, A) \subseteq \tilde{X} \setminus (F, A)$. Again, (G, A) is an $\tilde{s}p_c$-open set of each of its soft points. But as (G, A)does not contain any soft point of (F, A), no soft point of (G, A)can be $\tilde{s}p_c$ -limit soft point of (F, A). Therefore, no soft point of (G, A) can belong to $\tilde{s}p_c D(F, A)$. This implies that $(G, A) \subseteq \tilde{X} \setminus \tilde{s}p_c D(F, A)$. Hence, it follows that $x_\alpha \in (G, A) \subseteq \tilde{X} \setminus (F, A) \cap \tilde{X} \setminus \tilde{s}p_c D(F, A) \subseteq \tilde{X} \setminus ((F, A) \cup \tilde{s}p_c\, D(F, A))$. Therefore, $(F, A) \cup \tilde{s}p_c D(F, A)$ is $\tilde{s}p_c$ -closed. Hence, $\tilde{s}p_c cl(F, A) \subseteq (F, A) \cup \tilde{s}p_c\, D(F, A)$. Thus, $\tilde{s}p_c cl(F, A) = (F, A) \cup \tilde{s}p_c\, D(F, A)$.

Some properties of $\tilde{s}p_c$-closure of soft sets are given in the following results:

Proposition 3.28. For any soft subset (F, A) of a soft topological space $\tilde{X}$. The following statements are true:

1. $\tilde{X} \setminus \tilde{s}p_c cl(F, A) = \tilde{s}p_c int(\tilde{X} \setminus (F, A))$.
2. $\tilde{s}p_c cl(F, A) = \tilde{X} \setminus \tilde{s}p_c int(\tilde{X} \setminus (F, A))$.
3. $\tilde{X} \setminus \tilde{s}p_c int(F, A) = \tilde{s}p_c cl(\tilde{X} \setminus (F, A))$.
4. $\tilde{s}p_c int(F, A) = \tilde{X} \setminus \tilde{s}p_c cl(\tilde{X} \setminus (F, A)$.

Proof. We shall prove only (1), because the other parts can be proved similarly.

1) For any soft point $x_\alpha \in SP\,(X\,)_A$, $x_\alpha \in \tilde{X} \setminus \tilde{s}p_c cl(F, A)$ if and only if $x_\alpha \notin \tilde{s}p_c cl(F, A)$) if and only if for each $(G, A) \in \tilde{s}p_c O(X\,)$ containing x_α, there is $(F, A) \cap (G, A) = \tilde{\emptyset}$ if and only if $x_\alpha \in (G, A) \subseteq \tilde{X} \setminus (F, A)$ if and only if $x_\alpha \in \tilde{s}p_c int(\tilde{X} \setminus (F, A))$.

Proposition 3.29. For soft subsets (F, A) and (G, A) over $\tilde{X}$, the following statements are true:

1. $\tilde{s}p_c cl(F, A)$ is an $\tilde{s}p_c$-closed set in $\tilde{X}$ containing (F, A).
2. $\tilde{s}p_c cl(F, A)$ is the smallest $\tilde{s}p_c$ -closed set in $\tilde{X}$ containing (F, A)
3. (F, A) is $\tilde{s}p_c$-closed set if and only if $(F, A) = \tilde{s}p_c cl(F, A)$, then, $\tilde{s}p_c cl(\tilde{s}p_c\, cl(F, A)) = \tilde{s}p_c cl(F, A)$.

4. $\tilde{s}p_c\, cl(\tilde{\emptyset}) = \tilde{\emptyset}$ and $\tilde{s}p_c\, cl(\tilde{X}) = \tilde{X}$.
5. $(F,A) \subseteq \tilde{s}p_c cl(F,A)$.
6. $If\ (F,A) \subseteq (G,A)\ then\ \tilde{s}p_c cl(F,A) \subseteq \tilde{s}p_c\, cl(G,A)$.
7. $\tilde{s}p_c\, cl(F,A) \cup \tilde{s}p_c\, cl(G,A) \subseteq \tilde{s}p_c\, cl((F,A) \cup (G,A))$.
8. $\tilde{s}p_c\, cl\Big((F,A) \cap (G,A)\Big) \subseteq \tilde{s}p_c\, cl(F,A) \cap \tilde{s}p_c\, cl(G,A)$

Proof. Obvious.

Generally, the equality in (7) and (8) does not hold as shown in the following examples.
Example 3.30: Considering the result in Example 3.20 that $\tilde{s}p_c int(F,A) \cap \tilde{s}p_c int(G,A) \neq \tilde{s}p_c int((F,A) \cap (G,A))$. Also by Proposition 3.28 (4) $\tilde{s}p_c\, cl\big((F,A) \cup (G,A)\big) = \tilde{X} \backslash \tilde{s}p_c int\big(G_1(n) \cap G_2(n)\big) = \tilde{X} \backslash \tilde{s}p_c int(\{0\} = \tilde{X} \backslash \tilde{\emptyset} = \tilde{X}$, where $(F,A) = \tilde{X} \backslash G_1(n)$ and $(G,A) = \tilde{X} \backslash G_2(n)$. But $\tilde{s}p_c\, cl(F,A) \cup \tilde{s}p_c\, cl(G,A) = \tilde{X}(\tilde{s}p_c int G_1(n) \cup \tilde{s}p_c int G_2(n)) = \tilde{X} \backslash \{0\}$ .Therefore$\tilde{s}p_c\, cl\big((F,A) \cup (G,A)\big) \neq \tilde{s}p_c\, cl(F,A) \cup \tilde{s}p_c\, cl(G,A)$.
Example 3.31. Considering the soft space (X,τ,A) as defined in Example 3.19 if (F,A), (G,A) and $(H,A) = (F,A) \cap (G,A)$ define as $F(x) = \begin{cases} \{a,c\} & if\ x = 0 \\ \{a,c\} & if\ x = 1 \end{cases}$ $G(x) = \begin{cases} \{a,b\} & if\ x = 0 \\ \{b,c\} & if\ x = 1 \end{cases}$, $H(x) = \begin{cases} \{a\} & if\ x = 0 \\ \{c\} & if\ x = 1 \end{cases}$ $\tilde{s}p_c cl(F,A) = \tilde{X}$ and $\tilde{s}p_c cl(G,A) = \tilde{X}$, then $\tilde{s}p_c cl(F,A) \cap \tilde{s}p_c cl(G,A) = \tilde{X}$ and $\tilde{s}p_c cl(H,A) = (H,A)$. It follows that $\tilde{s}p_c cl(F,A) \cap \tilde{s}p_c cl(G,A) \neq \tilde{s}p_c cl((F,A) \cap (G,A))$.

Proposition 3.32. For a soft subset (F,A) over a soft space $\tilde{X}$, $\tilde{s}pcl\ (F,A) \subseteq \tilde{s}p_c\, cl(F,A)$.

Proof. Follows from the fact that every $\tilde{s}p_c$-closed set is soft pre-closed.

The following example shows that the converse of Proposition 3.32 does not hold in general.

Example3.33: Consider the soft topological space in Example 3.22 and take (F_2,A) then $\tilde{s}pcl(F_2,A) = (F_2,A)$ since (F_2,A) is soft pre closed but $\tilde{s}p_c cl(F_2,A) = (F,A)$ which $\tilde{s}pcl\ (F,A) \not\supseteq \tilde{s}p_c\, cl(F,A)$.

Corollary 3.34. Let (F,A) be any soft set of a soft space $\tilde{X}$. If $(F,A)$is both soft open and soft closed, then $(F,A) = \tilde{s}p_c\, int(\tilde{s}p_c\, cl(F,A))$.

Proof. Obvious.

Proposition 3.35. Let (Y,τ_Y,A) be a soft subspace of a soft space (X,τ,A) and $(F,A) \subset \tilde{Y}$. If $\tilde{Y}$ is soft clopen, then $\tilde{s}p_c\, clY\ (F,A) = \tilde{s}p_c\, cl(F,A) \cap \tilde{Y}$.

Proof. Let $x_\alpha \in \tilde{s}p_c\, cl(F,A) \cap \tilde{Y}$, then $x_\alpha \in \tilde{s}p_c\, cl(F,A)$ and $x_\alpha \in \tilde{Y}$. Take any $(G,A) \in \tilde{s}p_c O(Y)$ containing x_α. Since $\tilde{Y}$ is soft clopen by Proposition 2.13 $(G,A) \in \tilde{s}p_c O(X)$ containing x_α and hence$(G,A) \cap (F,A) \neq \tilde{\emptyset}$. This implies that $x_\alpha \in \tilde{s}p_c cl_Y(F,A)$ Thus $\tilde{s}p_c\, cl(F,A) \cap \tilde{Y} \subseteq \tilde{s}p_c cl_Y(F,A)$. Let $x_\alpha \in \tilde{s}p_c cl_Y(F,A)$, so that $x_\alpha \in \tilde{Y}, let\ (G,A) \in \tilde{s}p_c O(X)$ containing x_α. Then

By Corollary 2.1.29 $(G,A)\cap\tilde{Y}\in\tilde{s}p_cO(Y)$. Then $(G,A)\cap(F,A)\neq\tilde{\emptyset}$. So $x_\alpha\in\tilde{s}p_c\,cl(F,A)$ which implies that $x_\alpha\in\tilde{s}p_c\,cl(F,A)\cap\tilde{Y}$. Therefore $\tilde{s}p_c cl_Y(F,A)\subseteq\tilde{s}p_c cl(F,A)\cap\tilde{Y}$. Thus $\tilde{s}p_c cl_Y(F,A)=\tilde{s}p_c cl(F,A)\cap\tilde{Y}$.

Definition 3.36. Let (F,A) be a soft subset of a soft space $\tilde{X}$, then the $\tilde{s}p_c$ -boundary of $(F,A)$is define as $\tilde{s}p_c cl(F,A)\setminus\tilde{s}p_c int(F,A)$ and is denoted by $\tilde{s}p_c Bd(F,A)$.

Proposition3.37 For any soft subset $(F,A)$of a soft space $\tilde{X}$, we have the following properties:

1. $\tilde{s}p_c cl(F,A)=\tilde{s}p_c int(F,A)\cup\tilde{s}p_c Bd(F,A)$.
2. $\tilde{s}p_c int(F,A)\cap\tilde{s}p_c Bd(F,A)=\tilde{\emptyset}$
3. 3) $\tilde{s}p_c Bd(F,A)=\tilde{s}p_c cl(F,A)\cap\tilde{s}p_c cl(\tilde{X}\setminus(F,A))$.
4. $(4)\tilde{s}p_c Bd(F,A)\ is\ \tilde{s}p_c-closed.$
5. (5) $\tilde{s}p_c Bd(F,A)=\tilde{s}p_c Bd(\tilde{X}\setminus(F,A))$.
6. (6) $\tilde{s}p_c Bd(\tilde{s}p_c Bd(F,A))\subseteq\tilde{s}p_c Bd(F,A)$.
7. (7)$\tilde{s}p_c Bd(\tilde{s}p_c int(F,A))\subseteq\tilde{s}p_c Bd(F,A)$.
8. (8) $\tilde{s}p_c Bd(\tilde{s}p_c\,cl(F,A))\subseteq\tilde{s}p_c Bd(F,A)$.
9. (9) $\tilde{s}p_c int(F,A)=(F,A)\setminus\tilde{s}p_c Bd(F,A)$.
10. (10) $\tilde{X}=\tilde{s}p_c int(F,A)\cup\tilde{s}p_c int(\tilde{X}\setminus(F,A))\cup\tilde{s}p_c Bd(F,A)$.
11. (11) $\tilde{X}\setminus\tilde{s}p_c Bd(F,A)=\tilde{s}p_c int(F,A)\cup\tilde{s}p_c int(\tilde{X}\setminus(F,A))$.

Proof. Straightforward

Remark 3.38. Let (F,A) and (G,A) be soft subsets of a soft space $\tilde{X}$, then, $(F,A)\subseteq(G,A)$ does not imply that either $\tilde{s}p_c Bd(F,A)\subseteq\tilde{s}p_c Bd(G,A)$ or $\tilde{s}p_c Bd(G,A)\subseteq\tilde{s}p_c Bd(F,A)$, as it is varied in the following example.

Example 3.39: Considering the soft space (X,τ,A) as defined in Example 3.22 if

we take $(F_6,A)\subseteq(F_4,A)$ then $\tilde{s}p_c Bd(F_6,A)=(F_4,A)$ and $\tilde{s}p_c Bd(F_4,A)=\tilde{\phi}$. Thus $\tilde{s}p_c Bd(F_6,A)\nsubseteq\tilde{s}p_c Bd(F_4,A)$.

Proposition 3.40. For any soft subset (F, A) of a soft space $\tilde{X}$, $\tilde{s}pBd\,(F,A)\subseteq\tilde{s}p_c Bd(F,A)$.

Proof. $Let\ x_\alpha\in\tilde{s}pBd\,(F,A)$ and (G,A) be any $\tilde{s}p_c$-open set containing x_α, then, (G,A) is a soft pre-open set and $\tilde{s}pBd\,(F,A)=\tilde{s}pcl\,(F,A)\cap\tilde{s}pcl\,(\tilde{X}\setminus(F,A))$ implies that $(G,A)\cap(F,A)\neq\tilde{\emptyset}$ and $(G,A)\cap\tilde{X}\setminus(F,A)\neq\tilde{\emptyset}$, and hence by Proposition 3.37(3), $x_\alpha\in\tilde{s}p_c Bd(F,A).Thus\ \tilde{s}pBd\,(F,A)\subseteq\tilde{s}p_c Bd(F,A)$.

In general, the converse of Proposition 3.40 may not be true, as shown in the following example:

Example 3.41:.Considering the soft space (X,τ,A) as define in Example.3.22, if we take (F_6,A) then $\tilde{s}p_c Bd(F_6,A)=(F_4,A)$ and $\tilde{s}pBd(F_6,A)=\tilde{\emptyset}$. This shows that $\tilde{s}p_c Bd(F,A)\nsubseteq\tilde{s}pBd(F,A)$.

Next, $\tilde{s}p_c$-open and $\tilde{s}p_c$-closed sets in terms of $\tilde{s}p_c$-boundary are characterized in the following result **:**

Proposition 3.42: For a soft subset (F, A) of a soft space $\tilde{X}$, the following statements are true:

1. (F, A) is both $\tilde{s}p_c$-open and $\tilde{s}p_c$-closed if and only if $\tilde{s}p_cBd(F,A) = \widetilde{\emptyset}$.
2. $(F,A) \in \tilde{s}p_cC(X)$ if and only if $\tilde{s}p_cBd(F,A) \subseteq (F,A)$.
3. If (F,A) is $\tilde{s}p_c$-closed, *then* $\tilde{s}p_cBd(F,A) = (F,A) \setminus \tilde{s}p_c int(F,A)$.
4. $(F,A) \in \tilde{s}p_cO(X)$ if and only if $\tilde{s}p_cBd(F,A) \subseteq \tilde{X} \setminus (F,A)$ that is, $(F,A) \cap \tilde{s}p_cBd(F,A) = \widetilde{\emptyset}$
5. (F,A) is $\tilde{s}p_c$-open if and only if $\tilde{s}p_cBd(F,A) = \tilde{s}p_c D(F,A)$.

Proof. (1) Suppose that $\tilde{s}p_cBd(F,A) = \widetilde{\emptyset}$, then, $\tilde{s}p_cCl(F,A) \setminus \tilde{s}p_c int(F,A) = \widetilde{\emptyset}$, implies that $\tilde{s}p_c cl(F,A) = \tilde{s}p_c int(F,A) = (F,A)$. Therefore, (F, A) is both $\tilde{s}p_c$-open and $\tilde{s}p_c$-closed set. Conversely, if (F, A) is both $\tilde{s}p_c$-open and $\tilde{s}p_c$-closed set, then $(F,A) = \tilde{s}p_c int(F,A) = \tilde{s}p_c cl(F,A)$ and hence $\tilde{s}p_cBd(F,A) = \tilde{s}p_c cl(F,A) \setminus \tilde{s}p_c int(F,A) = \widetilde{\emptyset}$.

The proof of the other parts follows easily.

By the following example we show that $\tilde{s}p_cBd(\tilde{s}p_cBd(F,A)) \neq \tilde{s}p_cBd(F,A)$

Example 3.43: Considering the soft space (X,τ,A) as defined in Example 3.22 if

we take (F_6,A) then $\tilde{s}p_cBd(F_6,A) = (F_4,A)$ and since (F_4,A) is both $\tilde{s}p_c - open$ and $\tilde{s}p_c - closed$ then $\tilde{s}p_cBd(F_4,A) = \tilde{\phi}$. Thus $\tilde{s}p_cBd\tilde{s}p_cBd(F_6,A) = \tilde{\phi}$. It follows that $\tilde{s}p_cBd\tilde{s}p_cBd(F,A) \neq \tilde{s}p_cBd(F,A)$

Proposition 3.44: If $(F,A) = \tilde{s}p_cO(X) \cup \tilde{s}p_cC(X)$, then $\tilde{s}p_cBd\tilde{s}p_cBd(F,A) \neq \tilde{s}p_cBd(F,A)$

Proof. Let $(F,A) = \tilde{s}p_cO(X) \cup \tilde{s}p_cC(X)$. For any subset (F, A) of $\tilde{X}$, we have $\tilde{s}p_cBd\tilde{s}p_cBd(F,A) = (\tilde{s}p_c cl(F,A) \cap \tilde{s}p_c cl(\tilde{X} \setminus (F,A)) \cap \tilde{s}p_c cl(\tilde{X} \setminus \tilde{s}p_c clBd(F,A))$. Since, $(F,A) \in \tilde{s}p_c O(X)$ (resp. $(F,A) \in \tilde{s}p_c C(X)$), by Proposition 3.42(2), $(F,A) \cap \tilde{s}p_cBd(F,A) = \tilde{\phi}$ (resp. by Proposition 3.42(3), $\tilde{s}p_cBd(F,A) \subseteq (F,A)$) and hence $\tilde{s}p_c cl(F,A) \subseteq \tilde{s}p_c cl(\tilde{X} \setminus \tilde{s}p_cBd(F,A)$ (resp., $\tilde{s}p_c cl(\tilde{X} \setminus (F,A)) \subseteq \tilde{s}pc\, cl(\tilde{X} \setminus \tilde{s}p_cBd(F,A))$. Thus, we obtain $\tilde{s}p_cBd(\tilde{s}p_cBd(F,A)) = \tilde{s}p_c cl(F,A) \cap \tilde{s}p_c cl(\tilde{X} \setminus (F,A)) = \tilde{s}p_cBd(F,A)$ by Proposition 3.37 (3). This completes the proof.

It is well known that if $(F,A) \cap (G,A) = \tilde{\phi}$ and (F,A) is $\tilde{s}p_c$ -open, then $(F,A) \cap \tilde{s}p_c cl(G,A) = \tilde{\phi}$. Using this fact, the proof of the following proposition is immediately obtained.

Proposition 3.45*:* Let (F,A) and (G,A) be soft subsets of a soft space $\tilde{X}$. If $(F,A) \cap (G,A) = \tilde{\phi}$ and (F,A) is $\tilde{s}p_c$ -open set, then, $(F,A) \cap \tilde{s}p_c Bd(G,A) = \tilde{\phi}$.

Proposition 3.46. Let $(F,A) \subseteq (G,A)$ and $(G,A) \in \tilde{s}p_c C(X)$.Then , $\tilde{s}p_c Bd(F,A) \subseteq (G,A)$.

Proof. Since, $(F,A) \subseteq (G,A)$ implies $(F,A) \cap (X \setminus (G,A)) = \tilde{\phi}$ and $(\tilde{X} \setminus (G,A)) \in \tilde{s}p_cO(X)$, then by Proposition 3.42(5), $(\tilde{X} \setminus (G,A)) \cap \tilde{s}p_c Bd(F,A) = \tilde{\phi}$ implies $\tilde{s}p_c Bd(F,A) \subseteq (G,A)$.

4. Almost $\tilde{s}p_c$-continuous

Definition 4.1: A soft mapping $f_{pu}: (X, \tau, A) \to (Y, \tau_Y, B)$ is called almost $\tilde{s}p_c$-continuous at a soft point $x_\alpha \in SP(X)_A$, if for each soft open set (G, B) of $\tilde{Y}$ containing $f_{pu}(x_\alpha)$, there exists an $\tilde{s}p_c$-open set (F, A) of $\tilde{X}$ containing x_α such that $f_{pu}(F, A) \subseteq \tilde{s}int\tilde{s}cl(G, B)$. If f_{pu} is almost $\tilde{s}p_c$-continuous at every soft point of $\tilde{X}$, then it is called almost $\tilde{s}p_c$-continuous mapping

Theorem 4.2 : For a mapping $f_{pu}: (X, \tau, A) \to (Y, \tau_Y, B)$, the following statements are equivalent:

1. f_{pu} is almost $\tilde{s}p_c$-continuous.
2. For each $x_\alpha \in \tilde{X}$ and each soft open set (G, B) of $\tilde{Y}$ containing $f(X)_\alpha$ there exist s a $\tilde{s}p_c$-open set (F, A) in $\tilde{X}$ containing x_α, such that $f((F, A)) \subseteq \tilde{s}scl(G, B)$.
3. For each $x_\alpha \in \tilde{X}$ and each soft regular open set (G, B) of $\tilde{Y}$ containing $f(X)_\alpha$ there exist s a $\tilde{s}p_c$-open set (F, A) in $\tilde{X}$ containing x_α, such that $f((F, A)) \subseteq (G, B)$.
4. For each $x_\alpha \in \tilde{X}$ and each soft δ-open set (G, B) of $\tilde{Y}$ containing $f(X)_\alpha$ there exist s a $\tilde{s}p_c$-open set (F, A) in $\tilde{X}$ containing x_α, such that $f((F, A)) \subseteq (G, B)$.

Proof. . 1 → 2. Let $x_\alpha \in \tilde{X}$ and let (G, B) be a soft open set of $\tilde{Y}$ containing $f(X)_\alpha$. By (1) there exists a $\tilde{s}p_c$-open set $(F, A)$of $\tilde{X}$ containing x_α, such that $f((F, A)) \subseteq \tilde{s}int\tilde{s}cl(G, B)$. Since (G, B) is soft open set and hence (G, B) is soft preopen set. By Lemma 2.14 $\tilde{s}int\tilde{s}cl(G, B) = \tilde{s}scl(G, B)$. Therefore $f((F, A)) \subseteq \tilde{s}scl(G, B)$.

2 → 3. Let $x_\alpha \in \tilde{X}$ and let (G, B) be any soft regular open set of $\tilde{Y}$ containing $f(X)_\alpha$. Then $(G, B)$is a soft open set of $\tilde{Y}$ containing $f(X)_\alpha$.By (2) there exist a $\tilde{s}p_c$-open set (F, A) of $\tilde{X}$ containing x_α such that $f((F, A)) \subseteq \tilde{s}scl(G, B)$. Since (G, B) is soft regular open and hence soft pre-open set . By Lemma 2. 14 $\tilde{s}scl(G, B) = \tilde{s}int\tilde{s}cl(G, B)$.Therefore $f((F, A)) \subseteq \tilde{s}int\tilde{s}cl(G, B)$. Since (G, B) is soft regular open, then $f((F, A)) \subseteq (G, B)$.

3 → 4 . Let $x_\alpha \in \tilde{X}$ and let (G, B) be any soft δ- open set of $\tilde{Y}$ containing $f(X)_\alpha$. Then for each $f(X)_\alpha \in (G, B)$, there exists a soft open set (H, B) containing $f(X)_\alpha$ such that $(H, B) \subseteq \tilde{s}int\tilde{s}cl(H, B) \subseteq (G, B)$. Since $\tilde{s}int\tilde{s}cl(H, B)$ is soft regular open set of $\tilde{Y}$ containing $f(X)_\alpha$. By (3) there exist a $\tilde{s}p_c$-open set (F, A) of $\tilde{X}$ containing x_α such that $f\big((F, A)\big) \subseteq \tilde{s}int\tilde{s}scl(G, B) \subseteq (G, B)$.

4 → 1. Let $x_\alpha \in \tilde{X}$ and let (G, B) be any soft open set of $\tilde{Y}$ containing $f(X)_\alpha$. Then $\tilde{s}int\tilde{s}scl(G, B)$ is soft δ- open set of $\tilde{Y}$ containing$f(X)_\alpha$. By (4) there exist a $\tilde{s}p_c$-open set (F, A) of $\tilde{X}$ containing x_α such that $f\big((F, A)\big) \subseteq \tilde{s}int\tilde{s}scl(G, B)$. Therefore $f_{pu}$is almost $\tilde{s}p_c$-continuous.

Theorem 4.3: Let $f_{pu}: (X, \tau, A) \to (Y, \tau_Y, B)$ is an almost $\tilde{s}p_c$-continuous mapping and (G, B) be any soft open subset of $\tilde{Y}$. If $x_\alpha \in \tilde{s}p_c cl f_{pu}^{-1}((G, B)) \setminus f_{pu}^{-1}(G, B))$, then $f(x)_\alpha \in \tilde{s}p_c cl(G, B)$.

Proof. Let $x_\alpha \in \tilde{X}$ such that $x_\alpha \in \tilde{s}p_c cl f_{pu}^{-1}((G, B)) \setminus f_{pu}^{-1}(G, B))$, and suppose $f(x)_\alpha \notin \tilde{s}p_c cl(G, B)$. Then there exists a $\tilde{s}p_c$-open set (H, B) containing $f(x)_\alpha$ such that $(G, B) \cap (H, B) = \emptyset$. Then $\tilde{s}cl(H, B) \cap (G, B) = \emptyset$ implies that $\tilde{s}int\tilde{s}cl(H, B) \cap (G, B) = \emptyset$ and $\tilde{s}int\tilde{s}cl(H, B)$ is soft regular open set. Since f_{pu} is almost $\tilde{s}p_c$-continuous , by Theorem 4.2 there exists a $\tilde{s}p_c$-open set (F, A) of $\tilde{X}$ containing x_α such that $f_{pu}((F, A)) \subseteq \tilde{s}int\tilde{s}cl(H, B)$. Therefore $f_{pu}((F, A)) \cap (G, B) = \emptyset$. However , since $x_\alpha \in \tilde{s}p_c cl f_{pu}^{-1}((G,B)) \setminus f_{pu}^{-1}(G,B)) \neq \emptyset$ for every $\tilde{s}p_c$-open set (F,A) of $\tilde{X}$ containing x_α. So that $f_{pu}((F,A)) \cap (G,B) \neq \emptyset$.WE have a contradiction. It follows that $f(x)_\alpha \in \tilde{s}p_c cl(G,B)$.

Theorem 4.4: For a mapping $f_{pu}: (X,\tau,A) \to (Y,\tau_Y,B)$, the following statements are equivalent:

1. f_{pu} is almost $\tilde{s}p_c$-continuous
2. $f_{pu}^{-1}(\tilde{s}int\tilde{s}cl(G,B))$is $\tilde{s}p_c$-open set in $\tilde{X}$, for each soft open set (G,B) of $\tilde{Y}$.
3. $f_{pu}^{-1}(\tilde{s}int\tilde{s}cl(K,B))$is $\tilde{s}p_c$-closed set in $\tilde{X}$, for each soft closed set (K,B) of $\tilde{Y}$.
4. $f_{pu}^{-1}((K,B))$is $\tilde{s}p_c$-closed set in $\tilde{X}$, for each soft regular closed set (K,B) of $\tilde{Y}$.
5. $f_{pu}^{-1}((G,B))$is $\tilde{s}p_c$-open set in $\tilde{X}$, for each soft regular open set (G,B) of $\tilde{Y}$.

Proof. $1 \to 2$. Let (G,B) be a soft open set of $\tilde{Y}$. We have to show that $f_{pu}^{-1}(\tilde{s}int\tilde{s}cl(G,B))$is $\tilde{s}p_c$-open set in $\tilde{X}$. Let $x_\alpha \in f_{pu}^{-1}(\tilde{s}int\tilde{s}cl(G,B))$. Then $f(X)_\alpha \in \tilde{s}int\tilde{s}cl(G,B)$, and $\tilde{s}int\tilde{s}cl(G,B)$ is a soft regular open set in Y . Since f_{pu} is almost $\tilde{s}p_c$-continuous. Then by Theorem 3.2, there exist a $\tilde{s}p_c$-open set $(F,A)$of $\tilde{X}$ containing x_α such that $f((F,A)) \subseteq \tilde{s}int\tilde{s}cl(G,B)$, which implies that $x_\alpha \in (F,A) \subseteq f_{pu}^{-1}(\tilde{s}int\tilde{s}cl(G,B))$. Therefore $f_{pu}^{-1}(\tilde{s}int\tilde{s}cl(G,B))$ is $\tilde{s}p_c$-open set in $\tilde{X}$.
$2 \to 3$. Let (K,B) be any soft closed set of $\tilde{Y}$. Then $\tilde{Y} \setminus (K,B)$ is a soft open set of $\tilde{Y}$. By (2) $f_{pu}^{-1}(\tilde{s}int\tilde{s}cl(\tilde{Y} \setminus (K,B)))$ is $\tilde{s}p_c$-open set in $\tilde{X}$ and $f_{pu}^{-1}(\tilde{s}int\tilde{s}cl(\tilde{Y} \setminus (K,B)))= f_{pu}^{-1}(\tilde{s}int(\tilde{Y} \setminus \tilde{s}int(K,B)))= f_{pu}^{-1}(\tilde{Y} \setminus \tilde{s}cl\tilde{s}int(K,B))=\tilde{X} \setminus f_{pu}^{-1}(\tilde{s}cl\tilde{s}int(K,B))$ is $\tilde{s}p_c$-open set in $\tilde{X}$. And hence $f_{pu}^{-1}(\tilde{s}cl\tilde{s}int(K,B))$ is $\tilde{s}p_c$-closed set in $\tilde{X}$.
$3 \to 4$. Let (K,B) be any soft regular closed set of $\tilde{Y}$. Then (K,B) is a soft closed set of $\tilde{Y}$. By (3) $f_{pu}^{-1}(\tilde{s}cl\tilde{s}int(K,B))$ is $\tilde{s}p_c$-closed set in $\tilde{X}$. Since $(K,B)$is soft regular closed set. Then $f_{pu}^{-1}(\tilde{s}cl\tilde{s}int(K,B))$=$f_{pu}^{-1}(K,B)$. Therefore $f_{pu}^{-1}(K,B)$ is $\tilde{s}p_c$-closed set in $\tilde{X}$.
$4 \to 5$. Let (G,B) be any soft regular open set of $\tilde{Y}$. Then $\tilde{Y} \setminus (G,B)$ is soft regular closed set of $\tilde{Y}$ and By (4), we have $f_{pu}^{-1}(\tilde{Y} \setminus (G,B))= \tilde{X} \setminus f_{pu}^{-1}(G,B)$ is $\tilde{s}p_c$-closed set in $\tilde{X}$ and hence $f_{pu}^{-1}(G,B)$ is $\tilde{s}p_c$-open set in $\tilde{X}$.
$5 \to 1$. Let $x_\alpha \in \tilde{X}$ and let (G,B) be any soft regular open set of $\tilde{Y}$ containing $f(X)_\alpha$. Then $x_\alpha \in f_{pu}^{-1}(G,B)$. By (**5**), we have $f_{pu}^{-1}(G,B)$ is $\tilde{s}p_c$-open set in $\tilde{X}$. Therefore we obtain that $f_{pu}\left(f_{pu}^{-1}(G,B)\right) \subseteq (G,B)$. Hence by Theorem 3.2. f_{pu} is almost $\tilde{s}p_c$-continuous.

Theorem 4.5: A mapping $f_{pu}: (X,\tau,A) \to (Y,\tau_Y,B)$ is almost $\tilde{s}p_c$-continuous if and only if $f_{pu}^{-1}((G,B)) \subseteq \tilde{s}p_c int\, f_{pu}^{-1}(\, \tilde{s}int\tilde{s}cl(G,B)$ for every soft pre-open set (G,B) of $\tilde{Y}$.

Proof: Let (G,B) be any soft pre-open set of $\tilde{Y}$. Then $(G,B) \subseteq \tilde{s}int\tilde{s}cl(G,B)$ and $\tilde{s}int\tilde{s}cl(G,B)$ is soft regular open set of $\tilde{Y}$. Since f_{pu} is almost $\tilde{s}p_c$-continuous, by Theorem 4.4, $f_{pu}^{-1}(\, \tilde{s}int\tilde{s}cl(G,B)$ is $\tilde{s}p_c$-open set of $\tilde{X}$ and hence we obtain that $f_{pu}^{-1}((G,B)) \subseteq f_{pu}^{-1}(\, \tilde{s}int\tilde{s}cl(G,B) = \tilde{s}p_c int\, f_{pu}^{-1}(\, \tilde{s}int\tilde{s}cl(G,B)$.

Conversely, Let (G,B) be any soft regular open set of $\tilde{Y}$.Then (G,B) is soft pre-open set of $\tilde{Y}$. By hypothesis, we have $f_{pu}^{-1}((G,B)) \subseteq \tilde{s}p_c int\, f_{pu}^{-1}(\, \tilde{s}int\tilde{s}cl(G,B)) = \tilde{s}p_c int\, f_{pu}^{-1}((G,B))$. Therefore

$f_{pu}^{-1}((G,B))$ is $\tilde{s}p_c$-open set of $\tilde{X}$ and hence by Theorem 4.4 f_{pu} is almost $\tilde{s}p_c$-continuous

Theorem 4.6: For a mapping $f_{pu}: (X,\tau,A) \to (Y,\tau_Y,B)$, the following statements are equivalent:

1. f_{pu} is almost $\tilde{s}p_c$-continuous.
2. $f_{pu}(\tilde{s}p_c cl(F,A)) \subseteq \tilde{s}cl_\delta(f_{pu}(F,A))$, for each soft subset $(F,A)$of $\tilde{X}$.
3. $\tilde{s}p_c cl f_{pu}^{-1}((G,B)) \subseteq f_{pu}^{-1}(\tilde{s}cl_\delta((G,B))$, for each soft subset $(G,B)$of $\tilde{Y}$.
4. $f_{pu}^{-1}((K,B))$ is $\tilde{s}p_c$-closed in $\tilde{X}$, for each soft δ-closed subset (K,B) of $\tilde{Y}$.
5. $f_{pu}^{-1}((H,B))$ is $\tilde{s}p_c$-open in $\tilde{X}$, for each soft δ-open subset (H,B) of $\tilde{Y}$.
6. $f_{pu}^{-1}(\tilde{s}int_\delta(G,B)) \subseteq \tilde{s}p_c int f_{pu}^{-1}((G,B))$, for each soft subset $(G,B)$of $\tilde{Y}$.
7. $\tilde{s}int_\delta f_{pu}((F,A)) \subseteq f_{pu}(\tilde{s}p_c int\,(F,A))$, for each soft subset $(F,A)$of $\tilde{X}$.

Proof. 1 → 2. Let (F,A) be any soft subset over $\tilde{X}$. Since $\tilde{s}cl_\delta f_{pu}((F,A))$ is soft δ-closed subset $\tilde{Y}$, it is denoted by $\cap\{(K_\alpha,A):(K_\alpha,A)\in \tilde{s}RC(\tilde{Y}),\alpha\in\Delta\}$ where Δ is an index set, we have $(F,A)\subseteq f_{pu}^{-1}(\tilde{s}cl_\delta(f_{pu}((F,A)) = f_{pu}^{-1}(\cap\{(K_\alpha,A):\alpha\in\Delta\}) = \cap\{f_{pu}^{-1}((K_\alpha,A)); \alpha\in\Delta\}$. By (1) Theorem 4.2 $f_{pu}^{-1}(\tilde{s}cl_\delta(f_{pu}((F,A))$ is $\tilde{s}p_c$-closed in $\tilde{X}$. Hence $\tilde{s}p_c cl(F,A)\subseteq f_{pu}^{-1}(\tilde{s}cl_\delta(f_{pu}((F,A))$. Therefore, we obtain $f_{pu}(\tilde{s}p_c cl(F,A))\subseteq \tilde{s}cl_\delta(f_{pu}(F,A))$.

2 → 3. Let (G,B) be any subset of $\tilde{Y}$. Then $f_{pu}^{-1}((G,B))$ is a soft subset of $\tilde{X}$. By (2) we have $f_{pu}(\tilde{s}p_c cl f_{pu}^{-1}((G,B))\,)\subseteq \tilde{s}cl_\delta(f_{pu}f_{pu}^{-1}((G,B))\,) = \tilde{s}cl_\delta(G,B)$. Hence $\tilde{s}p_c cl f_{pu}^{-1}((G,B))\subseteq f_{pu}^{-1}(\tilde{s}cl_\delta(G,B))$.

3 → 4. Let (K,B) be a soft δ-closed subset of $\tilde{Y}$.By (3) $\tilde{s}p_c cl f_{pu}^{-1}((K,B))\subseteq f_{pu}^{-1}(\tilde{s}cl_\delta(K,B)) = f_{pu}^{-1}((K,B))$ and hence $f_{pu}^{-1}((K,B))$ is $\tilde{s}p_c$-closed in $\tilde{X}$.

4 → 5. Let (H,B) be a soft δ-open subset of $\tilde{Y}$. Then $\tilde{Y}\setminus(H,B)$ is a soft δ-closed subset of $\tilde{Y}$ and by (4) $f_{pu}^{-1}(\tilde{Y}\setminus(H,B)) = \tilde{X}\setminus f_{pu}^{-1}(H,B)$ is $\tilde{s}p_c$-closed in $\tilde{X}$. Hence $f_{pu}^{-1}(H,B)$ is $\tilde{s}p_c$-open in $\tilde{X}$.

5 → 6. For each soft subset $(G,B)$of $\tilde{Y}$. We have $\tilde{s}int_\delta(G,B)\subseteq(G,B)$. Then $f_{pu}^{-1}(\tilde{s}int_\delta(G,B))\subseteq f_{pu}^{-1}((G,B))$. By (5) $f_{pu}^{-1}(\tilde{s}int_\delta(G,B))$ is $\tilde{s}p_c$-open in $\tilde{X}$.Then $f_{pu}^{-1}(\tilde{s}int_\delta(G,B))\subseteq \tilde{s}p_c int f_{pu}^{-1}((G,B))$.

6 → 7. Let (F,A) be a soft subset of $\tilde{X}$. Then $f_{pu}((F,A)$ is a soft subset of $\tilde{Y}$. By (6) we obtain that $f_{pu}^{-1}(\tilde{s}int_\delta f_{pu}((F,A)))\subseteq \tilde{s}p_c int f_{pu}^{-1}(f_{pu}((F,A)))$. Hence $f_{pu}^{-1}(\tilde{s}int_\delta f_{pu}((F,A)))\subseteq \tilde{s}p_c int(F,A)$ which implies that $\tilde{s}int_\delta f_{pu}((F,A))\subseteq f_{pu}(\tilde{s}p_c int\,(F,A))$.

7 → 1. Let $x_\alpha\in\tilde{X}$ and let (G,B) be any soft regular open set of $\tilde{Y}$ containing $f(X)_\alpha$. Then $x_\alpha\in f_{pu}^{-1}(G,B)$. and $f_{pu}^{-1}(G,B)$ is a soft subset of $\tilde{X}$. By (7) we get $\tilde{s}int_\delta f_{pu}(f_{pu}^{-1}(G,B))\subseteq f_{pu}(\tilde{s}p_c int\, f_{pu}^{-1}(G,B))$ implies that $\tilde{s}int_\delta(G,B))\subseteq f_{pu}(\tilde{s}p_c int\, f_{pu}^{-1}(G,B)))$. Since (G,B) be a soft regular open set and hence is a soft δ-open set, then $(G,B))\subseteq f_{pu}(\tilde{s}p_c int\, f_{pu}^{-1}(G,B)))$, this implies that $f_{pu}^{-1}((G,B))\subseteq \tilde{s}p_c int\, f_{pu}^{-1}((G,B)$. Therefore $f_{pu}^{-1}((G,B))$ is $\tilde{s}p_c$-open in $\tilde{X}$ which contains x_α. So $f_{pu}$is almost $\tilde{s}p_c$-continuous.

Theorem 4.7: The set of all soft points x_α of $\tilde{X}$ at which $f_{pu}: (X,\tau,A)\to(Y,\tau_Y,B)$ is not almost $\tilde{s}p_c$-continuous is identical with the union of the $\tilde{s}p_c$-boundaries of the inverse image of soft regular open subsets of $\tilde{Y}$ containing $f_{pu}(x)_\alpha$.

Proof. If $f_{pu}$is not almost $\tilde{s}p_c$-continuous at x_α of $\tilde{X}$, then there exists a soft regular open set (G,B)

containing $f_{pu}(x)_\alpha$ such that for every $\tilde{s}p_c$-open set (F,A) of $\tilde{X}$ containing x_α, $f_{pu}\big((F,A)\big) \cap (\tilde{Y} \setminus (G,B)) \neq \phi$. This means that for every $\tilde{s}p_c$-open set (F,A) of $\tilde{X}$ containing x_α, we must have $\big((F,A)\big) \cap (\tilde{X} \setminus f_{pu}^{-1}(G,B)) \neq \phi$. Hence, it follows that $x_\alpha \in \tilde{s}p_c cl(\tilde{X} \setminus f_{pu}^{-1}\big((G,B)\big))$. But $x_\alpha \in f_{pu}^{-1}\big((G,B)\big)$. And hence $x_\alpha \in \tilde{s}p_c cl f_{pu}^{-1}\big((G,B)\big)$. This means that x_α belongs to the $\tilde{s}p_c$-boundaries of $f_{pu}^{-1}\big((G,B)\big)$.

Conversely, suppose that x_α belongs to the $\tilde{s}p_c$-boundary of $f_{pu}^{-1}\big((G_1,B)\big)$ for some soft regular open subset (G_1,B) of $\tilde{Y}$ such that $f_{pu}(x)_\alpha \in (G_1,B)$. Suppose that f_{pu} is almost $\tilde{s}p_c$-continuous mapping at x_α. Then by Theorem 3.2 there exists a $\tilde{s}p_c$-open set (F,A) of $\tilde{X}$ containing x_α such that $f_{pu}\big((F,A)\big) \subseteq (G_1,B)$. Then $(F,A) \subseteq f_{pu}^{-1}\big((G_1,B)\big)$. This shows that $x_\alpha \in \tilde{s}p_c int f_{pu}^{-1}\big((G,B)\big)$. Therefore, we have $x_\alpha \notin \tilde{s}p_c cl(\tilde{X} \setminus f_{pu}^{-1}\big((G,B)\big))$ and $x_\alpha \notin \tilde{s}p_c Bd f_{pu}^{-1}\big((G,B)\big)$. But this is a contradiction. This means that $f_{pu}$is not almost $\tilde{s}p_c$-continuous.

Proposition 4.8: A mapping $f_{pu}: (X,\tau,A) \to (Y,\tau_Y,B)$ is almost $\tilde{s}p_c$-continuous. If for each $x_\alpha \in SP(X)_A$, there exists a soft clopen set (F,A) of $\tilde{X}$ containing x_α such that $f_{pu}|(F,A); (F,A) \to \tilde{Y}$ is almost $\tilde{s}p_c$-continuous.

Proof. Let $x_\alpha \in SP(X)_A$, then by hypothesis, there exists a soft clopen set (F,A) of $\tilde{X}$ containing x_α such that $f_{pu}|(F,A); (F,A) \to \tilde{Y}$ is almost $\tilde{s}p_c$-continuous. Let (G,B) be any soft sub set of $\tilde{Y}$ containing $f(x)_\alpha$, there exists a $\tilde{s}p_c$-open subset (H,A) of (F,A) containing x_α such that $\big(f_{pu}|(F,A)\big)(H,A) \subseteq \tilde{s}int\tilde{s}cl(G,B))$. Since (F,A) is soft clopen set. By Proposition 2.12 (H,A) is $\tilde{s}p_c$-open subset in $\tilde{X}$ and hence $f_{pu}(H,A) \subseteq \tilde{s}int\tilde{s}cl(G,B))$. This implies that f_{pu} is almost $\tilde{s}p_c$-continuous.

Theorem 4.9: . If $\tilde{X} = (F,A) \cup (G,A)$, where (F,A) and (G,A) are soft clopen sets and $f_{pu}: (X,\tau,A) \to (Y,\tau_Y,B)$ is a mapping such that both $f_{pu}|(F,A)$ and $f_{pu}|(G,A)$ are both almost $\tilde{s}p_c$-continuous, then f_{pu} is almost $\tilde{s}p_c$-continuous.

Proof. Let (H,B) be any soft regular open set of $\tilde{Y}$. Then $f_{pu}^{-1}((H,B)) = (f_{pu}|(F,A))^{-1}((H,B)) \cup (f_{pu}|(G,A))^{-1}((H,B))$. Since $f_{pu}|(F,A)$ and $f_{pu}|(G,A)$ is almost $\tilde{s}p_c$-continuous. Then by Theorem 4.4 $(f_{pu}|(F,A))^{-1}((H,B))$ and $(f_{pu}|(G,A))^{-1}((H,B))$ are $\tilde{s}p_c$-open sets in (F,A) and (G,A) respectively. Since (F,A) and (G,A) are soft clopen sets in $\tilde{X}$, then by Proposition .2.12 (Al-kadi, 2014) (Placeholder1) $(f_{pu}|(F,A))^{-1}((H,B))$ and $(f_{pu}|(G,A))^{-1}((H,B))$ are $\tilde{s}p_c$-open sets in $\tilde{X}$. Since the union of two $\tilde{s}p_c$-open sets is $\tilde{s}p_c$-open . Hence $f_{pu}^{-1}((H,B))$ is $\tilde{s}p_c$-open sets in $\tilde{X}$. Therefore by Theorem 4.4 f_{pu} is almost $\tilde{s}p_c$-continuous.

In general, if $\tilde{X} = \cup \{(K_\alpha,A);\ \alpha \in \Delta\}$, where each (K_α,A) is a soft clopen set and $f_{pu}: (X,\tau,A) \to (Y,\tau_Y,B)$ is a mapping such that $f_{pu}|(K_\alpha,A)$ is almost $\tilde{s}p_c$-continuous for each α, then f_{pu} is almost $\tilde{s}p_c$-continuous.

Theorem 4.10: Let $\tilde{X} = (F_1,A) \cup (F_2,A)$, where (F_1,A) and (F_2,A) are soft clopen sets . Let

$f_{pu}:(F_1,A) \rightarrow (Y,\tau_Y,B)$ and $g_{pu}:(F_2,A) \rightarrow (Y,\tau_Y,B)$ be almost $\tilde{s}p_c$-continuous. If $f_{pu}(x)_\alpha = g_{pu}(x)_\alpha$ for each $x_\alpha \in (F_1,A) \cap (F_2,A)$. Then the mapping $h_{pu}:(F_1,A) \cap (F_2,A) \rightarrow \tilde{Y}$ such that $h_{pu}(x)_\alpha = \begin{cases} f_{pu}(x)_\alpha & if\ x_\alpha \in (F_1,A) \\ g_{pu}(x)_\alpha & if x_\alpha \in (F_2,A) \end{cases}$ is almost $\tilde{s}p_c$-continuous.

Proof. Let (G,B) be a soft regular open set of $\tilde{Y}$. Now $h_{pu}^{-1}\big((G,B)\big) = f_{pu}^{-1}((G,B)) \cup g_{pu}^{-1}((G,B))$. Since $f_{pu}$is almost $\tilde{s}p_c$-continuous, then by Theorem 4.4, $f_{pu}^{-1}((G,B))$ is $\tilde{s}p_c$-open set in (F_1,A). But (F_1,A) is soft clopen set in $\tilde{X}$. Then by Proposition 2.12 $f_{pu}^{-1}((G,B))$ is $\tilde{s}p_c$-open set in $\tilde{X}$. Similarly $g_{pu}^{-1}((G,B))$ is $\tilde{s}p_c$-open set in (F_2,A), and hence $\tilde{s}p_c$-open set in $\tilde{X}$. Since the union of two $\tilde{s}p_c$-open set is $\tilde{s}p_c$-open. Therefore, $h_{pu}^{-1}\big((G,B)\big) = f_{pu}^{-1}((G,B)) \cup g_{pu}^{-1}((G,B))$ is $\tilde{s}p_c$-open set in $\tilde{X}$. Hence by Theorem 4.4 h_{pu} is almost $\tilde{s}p_c$-continuous.

Theorem 4.11: Let $f = f_{p_1u_1}:(X,\tau,A) \rightarrow (Y,\tau_Y,B)$ be almost $\tilde{s}p_c$-continuous and $g = g_{p_2u_2}:(Y,\tau_Y,B) \rightarrow (Z,\tau_Z,C)$ is soft continuous and soft open mapping. Then, the soft composition mapping $g \circ f:(X,\tau,A) \rightarrow (Z,\tau_Z,C)$ is almost $\tilde{s}p_c$-continuous.

Proof. Let $x_\alpha \in SP(X)_A$ and (G,C) be any soft open set of $\tilde{Z}$ containing $g(f(x)_\alpha)$. Since g is soft continuous, $g^{-1}((G,C))$ is a soft open set in $\tilde{Y}$ containing $f(x)_\alpha$. Since f is almost $\tilde{s}p_c$-continuous, there exists a $\tilde{s}p_c$-open set (F,A) of $\tilde{X}$ containing x_α such that $f\big((F,A)\big) \subseteq \tilde{s}int\tilde{s}clg^{-1}((G,C))$. Also, since g is soft continuous, then we obtain that $g \circ f\big((F,A)\big) \subseteq g(\tilde{s}intg^{-1}(\tilde{s}cl\big((G,C)\big)))$. Since g is a soft open, we obtain $g \circ f\big((F,A)\big) \subseteq \tilde{s}int\tilde{s}cl\big((G,C)\big)$. Therefore $g \circ f$ is soft almost $\tilde{s}p_c$-continuous.

Finally we give the relation between $\tilde{s}p_c$-continuous, almost $\tilde{s}p_c$-continuous almost soft continuous and soft continuous mapping.

Theorem 4.12: Let $f_{pu}:(X,\tau,A) \rightarrow (Y,\tau_Y,B)$ is an almost $\tilde{s}p_c$-continuous and $\tilde{Y}$ is soft semi regular. Then $f_{pu} is\ \tilde{s}p_c$-continuous.

Proof. Let $x_\alpha \in SP(X)_A$ and (G,B) be any soft open set of $\tilde{Y}$ containing $f_{pu}(x_\alpha)$. By the soft semi regularity of $\tilde{Y}$, there exists a soft regular open set (H,B) of $\tilde{Y}$ such that $f_{pu}(x_\alpha) \in (H,B) \subseteq (G,B)$. Since f_{pu} is almost $\tilde{s}p_c$-continuous. By Theorem 4.2 there exist a $\tilde{s}p_c$-open set $(F,A)$of $\tilde{X}$ containing x_α such that $f_{pu}((F,A)) \in (H,B) \subseteq (G,B)$. Therefore $f_{pu}\ is\ \tilde{s}p_c$-continuous.

Theorem 4.13: If (Y,τ_Y,B) is a soft hyperconnected space, then every soft mapping $f_{pu}:(X,\tau,A) \rightarrow (Y,\tau_Y,B)$ is an almost $\tilde{s}p_c$-continuous.

Proof. Let $x_\alpha \in SP(X)_A$ and (G,B) be any soft open set of $\tilde{Y}$ containing $f_{pu}(x_\alpha)$. Since $\tilde{Y}$ is a soft hyperconnected space. Then $\tilde{s}cl(G,B) = \tilde{Y}$ and hence $\tilde{s}int\tilde{s}cl\big((G,C)\big) = \tilde{Y}$. Therefore, we have $f_{pu}((F,A)) \subseteq \tilde{s}int\tilde{s}cl\big((G,C)\big)$, where (F,A) be any $\tilde{s}p_c$-open ser in $\tilde{X}$. This show that f_{pu} is an almost $\tilde{s}p_c$-continuous**.**

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