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Statical Nonlinear Analysis of Spherical Assemblies Utilizing Pade Approximation

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Abstract:
A new numerical technique for computing the displacement and internal force is presented and applied to the double-layer spherical model. This numerical approach takes into consideration the geometrical nonlinear response of the pin-jointed rigid systems. The presented method performs a practical way of employing the large deformation within the elastic limit for analyzing space structures. In the proposed technique the nonlinear geometrical response of the assembly is modeled and analyzed as a system of algebraic nonlinear equations. The Pade approximation method is conducted in the derivation to give a high rate of convergent ratio in solving the nonlinear equations. The result is validated using the nonlinear finite element software SAP2000 and the linear force method. The discrepancies between the proposed technique and SAP2000 analysis results for external nodal displacement difference and internal element force difference are computed and compared with the linear technique outcomes. The Euclidean norm index is also used to test the precision of calculated nonlinear nodal displacements. The findings showed more closeness to nonlinear SAP2000 results than the linear method.

Keywords: *Geometric Nonlinearity; Nonlinear Analysis; Statical Analysis; Spherical; Force Method.*

1. Introduction

Spheres are always considered unique and elegant geometry for structures [1, 2]. Architects and engineers have built a variety of spherical forms all over the world. Many spherical buildings can be seen as landmarks, such as Al Wasl Plaza in Dubai in UAE as shown in Figure 1 [3]. Due to the geometrical characteristics of spheres, they are used to afford a wide span as a lightweight structure with economical choice. When they are affected by specific external loads, they face notable deformation [4, 5]. Therefore, they require a very precise computation during the analysis and design process considering their geometrical nonlinear behavior [6].

Many nonlinear analysis techniques have been established for analyzing the nonlinear static and dynamic responses of structures. In the early stages when researchers considered the geometric nonlinearity behavior, they applied techniques of incremental stiffness, Newton Raphson, and iteration procedure [7]. The dynamic relaxation method is one of the popular methods conducted in dealing with geometrically nonlinear static analysis at a steady state [8-13]. Improving the tangent stiffness matrix in the finite element method is another way for performing nonlinear analysis with geometrical consideration [14-17]. Minimum potential energy is an additional different method depending on minimizing the total potential energy of the entire set to provide the equilibrium state [18-21]. The further analysis technique is the nonlinear force method (NFM). In this approach, the three basic principles which are equilibrium, compatibility, and constitutive relationships are adopted. The geometrical nonlinearity can be computed by extending the linear force method [5, 22] to be used in the iteration procedure [23, 24], or be used in the specific algorithm [25]. Additionally, Manguri and Saeed [26] and Saeed et al. [27] presented an approximate linear force method working on updating joint coordinates of the geometrical nodes for the iteration steps using the discretized applied load. Kwan [6] proposed a new technique for analyzing prestressed cable systems using the Taylor series for expressing geometrical nonlinearity within both the compatibility condition and equilibrium state. As a result, the derivation of compatibility and equilibrium matrices came out in a deformed configuration. Most of the quoted analysis methods require a very well understanding and regularly work with a specific written algorithm or iteration process, which may be time-consuming, or sometimes fail in analyzing complex structures [6, 28].

In the article, an alternative approach is conducted in analyzing the spherical model which is a nonlinear geometrical structure. The proposed technique is based on the nonlinear force method expanded using the Pade approximation method. Previously, the Pade approximation method is used in prestressing spatial pin-jointed structures by indicating nonlinear member actuation to provide the required degree of prestressed force using the flexibility method [29]. This work is carried out to show the significance of considering the geometrical nonlinearity behavior for structures experiencing great deformability within elastic limits. In addition, the nonlinear finite element analysis SAP2000 software is used to validate the proposed technique. Finally, the comparison is made between the nonlinear techniques and the linear force method to show the necessity of utilizing nonlinear approaches for structures that have geometrical nonlinearity.

The outline of this paper is arranged as the following. Section 1 briefly introduces nonlinear analysis techniques and geometric nonlinearity behavior. Section 2 is the formulation of the proposed technique. In Section 3 the numerical example of the double-layer spherical model is analyzed by the proposed technique and SAP2000 software. Finally, the conclusions of the analysis findings are presented in Section 4.

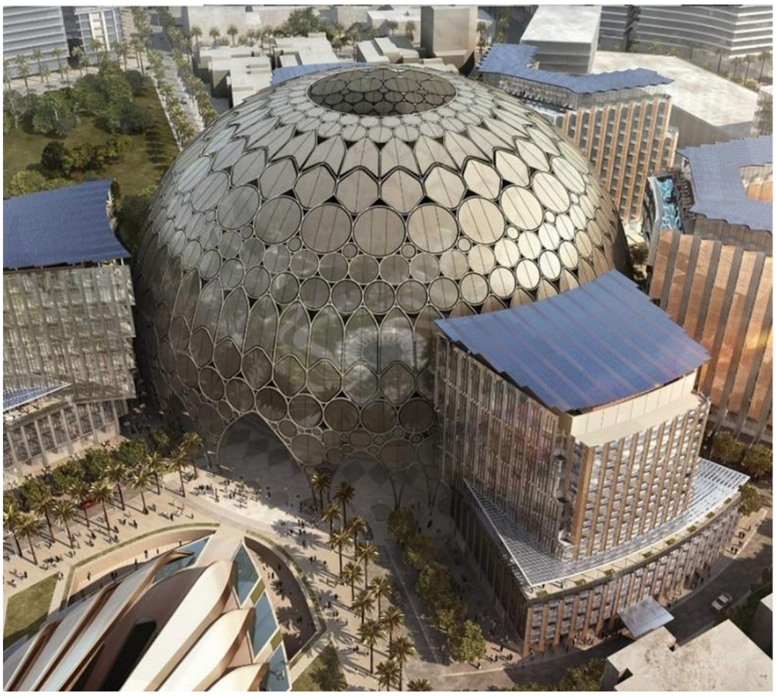

Figure 1: Al Wasl plaza spherical structure[3].

2. Geometrical Nonlinear Force Method

This technique is derived depending on the relation between the exterior nodal displacement and the member variation length of the bar element with end nodes 1 and 2 prior to the loading. The nodes of the bar element 1-2 displace to 1′-2′ after loading as shown in Figure 2.

When an assembly is affected by nodal external loads P, it is required to stay at equilibrium with internal member force t and the relation between them can be shown as below:

(1)
$$\boldsymbol{Q}(\boldsymbol{d})\boldsymbol{t} = \boldsymbol{P}$$

Similarly, the geometrical deformability which is between the external joint displacements d and internal member alteration e can be expressed as:

(2)
$$\boldsymbol{C}(\boldsymbol{d})\boldsymbol{d} = \boldsymbol{e}(\boldsymbol{d})$$

where $\boldsymbol{Q}(\boldsymbol{d})$ and $\boldsymbol{C}(\boldsymbol{d})$ are the equilibrium and compatibility matrices after geometrical deformation, and $\boldsymbol{Q}(\boldsymbol{d}) = \boldsymbol{C}^T(\boldsymbol{d})$ [30].

Since the assumption of a constitutive relationship has no impact on the equilibrium equation and compatibility condition, the relation between the matrix of member forces and vector of member length alteration can be formulated as:

(3)
$$\boldsymbol{e}(\boldsymbol{d}) - \boldsymbol{Ft} = 0$$

where F is the flexibility matrix. The variation of the member length can be obtained from $e = L' - L$, as well, $L' = \sqrt{\left(x_o + dx_o\right)^2 + \left(y_o + dy_o\right)^2}$ as shown in Figure 2. Where L is the initial member length, L′ is the new length of the member and the notation ( )o = ( )2 – ( )1 as shown in Figure 2.

Now employing the Pade approximation method that is a very powerful mathematical technique to attain the numerical solution for e. The Pade approximation is a conventional rational function whose extension is pointed to settle with Taylor's series expansion of the main function as distant as conceivable. In most cases, the Pade approximation affords a more improved approximation for the original function and could work where Taylor's series does not converge. Thus, e can be expanded and expressed as below [29, 31-33]:

(4)
$$e = L \times \left\{ \left(\frac{4 + \dfrac{3\left(2x_o dx_o + 2y_o dy_o + dx_{ji}^2 + dy_{ji}^2\right)}{L^2}}{4 + \dfrac{\left(2x_o dx_o + 2y_o dy_o + dx_o^2 + dy_o^2\right)}{L^2}} \right) - 1 \right\}$$

Once more, considering the large deflection, the equilibrium state between applied load P_x and P_y and the internal axial force t can be given as:

(5)
$$\begin{aligned} P_x &= \pm\, t \times \cos\theta \\ P_y &= \pm\, t \times \sin\theta \end{aligned}$$

where $\cos\theta = \dfrac{4x_o L^2 + 4dx_o L^2 + 2x_o^2 dx_o + 2x_o y_o dy_o}{4L^3 + 6L\left(x_o dx_o + y_o dy_o\right)}$, and similarly

$\sin\theta = \dfrac{4y_o L^2 + 4dy_o L^2 + 2y_o^2 dy_o + 2y_o x_o dx_o}{4L^3 + 6L\left(x_o dx_o + y_o dy_o\right)}$.

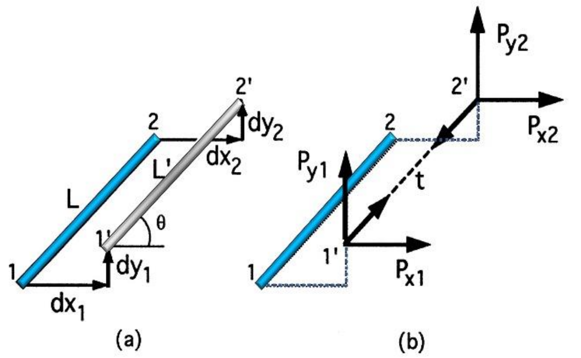

Figure 2: (a) Original and deformed length of element 1-2 (b) Original and deformed equilibrium state of element 1-2

Now by substituting (4) into (3) we get a system of the nonlinear equations that can be solved using any nonlinear solver technique to find out the nodal displacement and internal force for nonlinear geometric analysis. Here, we used the fsolve in MATLAB to employ the solution. Later, the Euclidean norm ratio is implemented to show the difference ratio of displacement (R_d) between the nonlinear and linear displacement as below:

(6) $$R_d = \frac{\left\| d_{nonlinear} - d_{linear} \right\|_2}{\left\| d_{linear} \right\|_2} \times 100$$

3. Numerical Example

For apprising the validity of the proposed technique, a very complicated structure is selected. The spherical double-layer model [1] shown in Figure 3 is analyzed using the technique presented in the previous section. The outer diameter of the sphere is 8m and the distance between both layers is 200mm. The model consists of 382 nodes, that 21 nodes of the outer layer from the bottom on a diameter of 1.174m are pin supports as shown in Figure 4. It has 1520 members. The axial stiffness of all the members is 15707963.3 MPa. The model is laterally loaded in the x-direction by 1000 N at 73 nodes on the outer surface (20-30, 42-50, 62-70, 82-90, 102-110, 122-130, 142-150, 162-170, 381) as shown in Figure 4. The lateral loads produced noticeable deformability in all of the x-, y-, and z-directions. The results are presented in the following section.

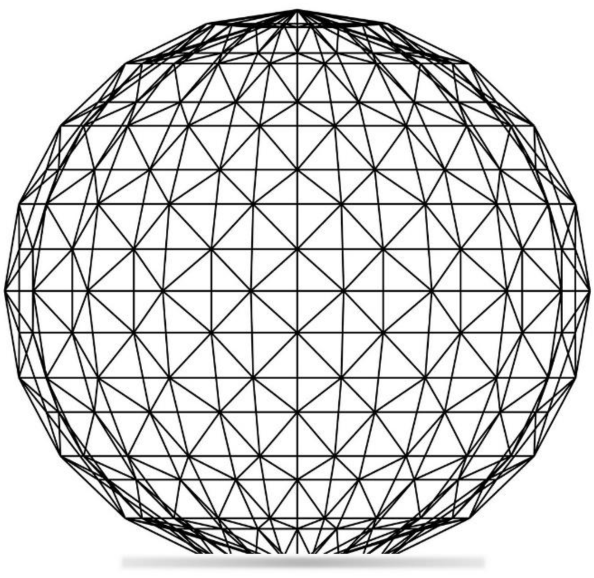

Figure 3: Double-layer spherical model

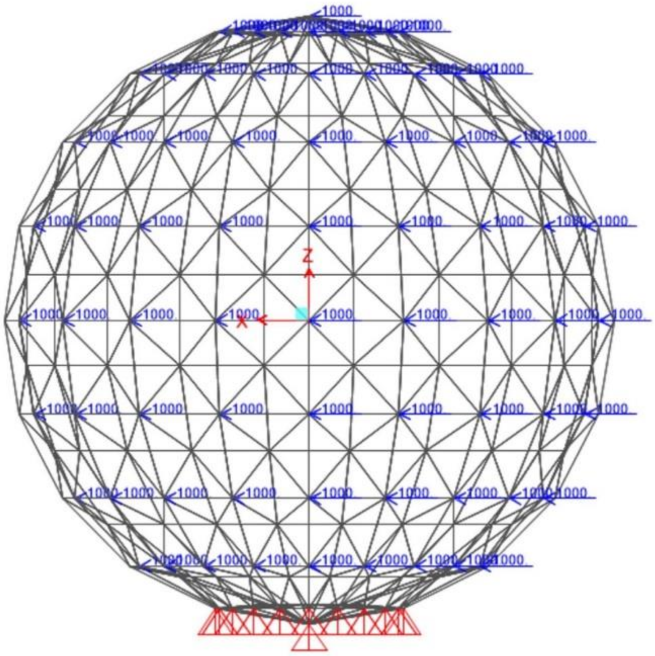

Figure 4: Laterally loaded in (N) double-layer spherical model

4. Result and Discussion

The double-layer spherical model is theoretically analysed using the presented approach. The maximum axial force for this specifically applied load shown in Figure 4 is located at members 862 as 45.456 kN, and 857 as -45.456 kN [1]. While the present technique determined tensile 45.282 kN and compressive 45.643 kN for members 862 and 857 respectively, which are almost equal to the SAP2000 results with values 45.281 kN, -45.642 kN for the same members. Likewise, to show the difference of maximum member forces between the present approach and SAP2000 analysis (SNF) and also between linear technique and SAP2000 findings (SLF) the force discrepancy is shown in Figure 5. The difference value between nonlinear techniques (SNF) is 0.001, while for SLF is 0.175 for member 862 and 0.186 for member 857. These similarities between nonlinear techniques are due to the impact of geometrical stiffness on the member force computation and vice versa.

The analysis results for the displaced selected nodes in the x-, y-, and z-directions via applying the current technique are presented in columns 2-4 of Table 1. Similarly, the model was analyzed by nonlinear finite element analysis using SAP2000. The findings are presented in columns 5-7 of Table 1. Both nonlinear analysis results are in very well agreement with each other. Later, they compared with linear analysis result by Mahmood, et al. [1] as shown in columns 8-10 of Table 1. The Euclidean norm ratio as in (4) is used to find out the difference rate between the nonlinear displacement and the linear displacement. The difference rate is about 0.11%, and this rate will increase when the model faces greater loading values for the same loading condition. Moreover, the difference in resultant displacements between the SAP2000 and present technique SND and also between SAP2000 and linear technique SLD are identified and presented in Figure 6. The difference between the nonlinear techniques SND is barely noticeable for all joints that its maximum amount is 0.002. While the difference of SLD is clearly visible that its maximum difference amount is 0.146 which is refer to neglecting the effect of geometric nonlinearity.

Considering most systems of geometrical nonlinear behavior response provide more precision outputs, particularly when they are experiencing large deformability.

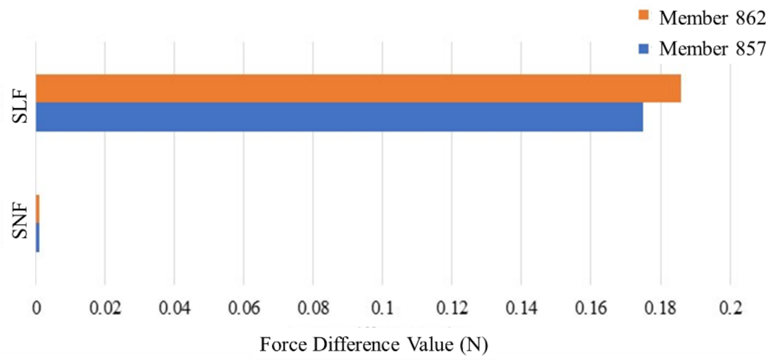

Figure 5: Difference of maximum member forces of SNF and SLF

Table 1: Nodal Displacement of Double-layer Spherical Model

Nodes	Present Technique (mm)			SAP2000 (mm)			Linear FM [1] (mm)		
	dx	*dy*	*dz*	*dx*	*dy*	*dz*	*dx*	*dy*	*dz*
1-20	0	0	0	0	0	0	0	0	0
25	11.358	-1.018	-6.250	11.358	-1.018	-6.250	11.38	-1.05	-6.05
30	8.998	2.336	18.533	8.998	2.336	18.534	8.85	2.37	18.64
35	7.197	0.172	5.817	7.197	0.172	5.817	7.17	0.25	6.04
40	7.308	-1.587	-18.715	7.308	-1.587	-18.716	7.45	-1.57	-18.59
45	23.494	-0.691	-10.299	23.494	-0.691	-10.299	23.54	-0.68	-10.08
50	22.454	2.758	30.767	22.454	2.758	30.768	22.26	2.81	31.00
55	16.828	1.279	9.746	16.828	1.279	9.747	16.78	1.38	10.04
60	19.757	-3.576	-31.181	19.757	-3.576	-31.181	19.93	-3.52	-30.90
70	34.887	3.567	36.626	34.888	3.568	36.627	34.67	3.65	36.93
80	31.494	-4.300	-37.191	31.494	-4.300	-37.192	31.69	-4.22	-36.84
90	47.886	4.000	38.591	47.886	4.000	38.591	47.66	4.12	38.96
95	41.137	1.403	12.228	41.138	1.403	12.229	41.06	1.52	12.65
100	44.200	-4.541	-39.362	44.201	-4.541	-39.362	44.40	-4.43	-38.94
105	63.144	-1.367	-12.450	63.145	-1.367	-12.450	63.19	-1.22	-12.04
110	60.772	3.907	36.623	60.774	3.907	36.624	60.56	4.05	37.06
115	54.336	1.293	11.570	54.336	1.294	11.571	54.26	1.44	12.06
120	57.216	-4.412	-37.603	57.217	-4.412	-37.604	57.40	-4.27	-37.12
125	74.310	-1.252	-10.691	74.312	-1.252	-10.691	74.35	-1.08	-10.22
130	72.188	3.362	30.946	72.190	3.362	30.947	72.01	3.53	31.44
135	66.665	1.075	9.699	66.667	1.075	9.699	66.60	1.25	10.24
140	69.163	-3.867	-32.075	69.165	-3.867	-32.076	69.31	-3.70	-31.54
145	82.627	-1.016	-7.927	82.628	-1.016	-7.927	82.65	-0.82	-7.40
150	81.001	2.439	22.185	81.002	2.439	22.185	80.86	2.63	22.73
155	76.954	0.745	6.800	76.955	0.745	6.800	76.90	0.94	7.39
160	78.835	-2.935	-23.343	78.837	-2.935	-23.343	78.94	-2.74	-22.76
165	87.320	-0.669	-4.435	87.322	-0.669	-4.435	87.32	-0.46	-3.86
170	86.371	1.238	11.259	86.373	1.238	11.259	86.29	1.44	11.84
175	84.205	0.288	3.218	84.207	0.288	3.219	84.16	0.49	3.82
180	85.201	-1.678	-12.411	85.203	-1.678	-12.411	85.24	-1.47	-11.81
382	87.747	-0.211	-0.601	87.749	-0.211	-0.601	87.73	0	0

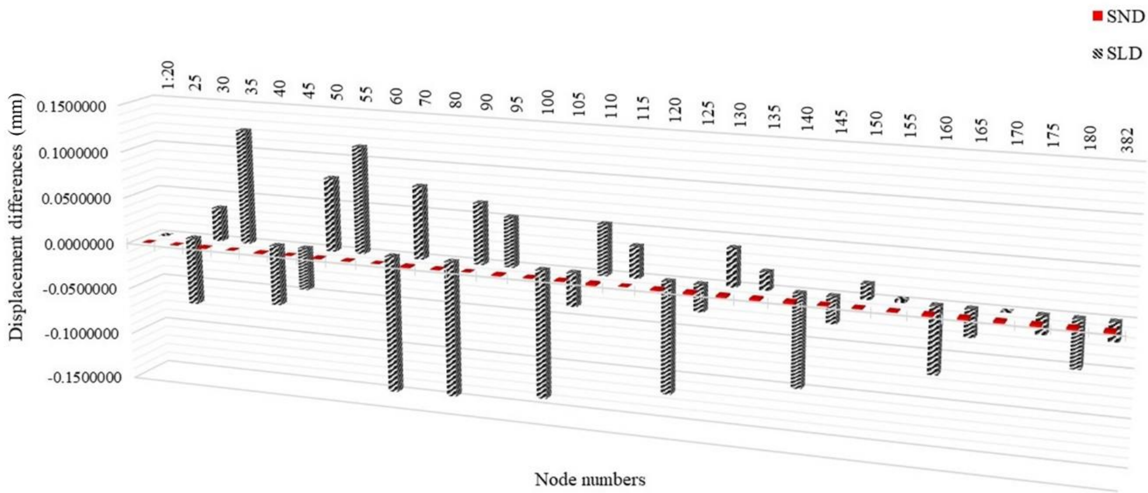

Figure 6: Differences of resultant displacement obtained through nonlinear and linear force methods with respect to SAP2000.

5. Conclusion

The newly derived nonlinear geometrical analysis force method is proposed and applied to the double-layer spherical structure. Likewise, the same model was analyzed using the nonlinear finite element software SAP2000 to verify the computed technique. The outcome displacements and maximum axial forces are compared with the linear technique. The rate of Euclidean norm index between the vectors of nonlinear and linear resultant joint displacements came out as 0.11%. That causes of minimizing the cost function due to geometrical stiffness consideration for that applied loading condition. The maximum discrepancy for both of the displacement and member force between nonlinear analysis results are 0.002 and 0.001 respectively. However, these discrepancy values came out as 0.146 for displacement differences, and 0.175 and 0.186 for tensile and compressive force differences when compared to the linear approach. The results showed the applicability of the technique in analyzing such a complex structure by concerning the nonlinear behavior of the structures. The employment of the Pade approximation method in expanding the nonlinear member variation and internal force components provided a very convergeable function in solving the nonlinear equations.

6. Conflict of Interest:

The authors have no conflicts of interest.

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