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On The Investigating Cycle Properties In The Galilean Plane G^2

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Abstract:
The introduction of the Galilean plane within the affine plane parallels the familiar concepts of the Euclidean plane, extending the realm of geometric exploration. The fundamental concepts of lines, triangles, squares, and circles are important in both planes, allowing for a smooth transition between these mathematical environments. The noteworthy aspect is the discovery that cycles in the Galilean plane have properties similar to circles in the Euclidean plane. This paper contributes to the mathematical literature by carefully deriving and establishing features of cycles in the Galilean plane, exhibiting their startling resemblance to Euclidean circles. The use of the inscribed angle as an alternative definition of the circle is particularly insightful, providing a faster and more intuitive explanation of some findings than the usual definition. Such comparative assessments not only broaden our understanding of various geometries but also give us chances to streamline the learning process. The paper argues for the inclusion of Galilean geometry in the high school curriculum by highlighting these parallels. It implies that exposing students to various geometrical systems not only broadens their mathematical perspectives but also fosters a larger and more inclusive vision of the subject, potentially inspiring increased interest and acknowledgment of Galilean geometry among students.

Keywords: *Cycle, E^2, G^2, Non-Euclidean Geometry, Special Line.*

1. Introduction

Researchers have looked into and discussed the subject of teaching and learning geometry to high school students on various platforms. Despite having many applications within and outside mathematics, geometry has limitations in teaching and learning at different educational levels. I'd like to dwell on the Galilean plane to make an exciting difference.

The fundamental feature of Galilean geometry is its relative simplicity, which allows students to study it in depth without wasting much time or intellectual energy. To put it another way, the simplicity of Galilean geometry facilitates its overall growth, and significant development of a new geometric system is required before it can be effectively compared to Euclidean geometry. Furthermore, a comprehensive outcome will likely give the learner psychological security and a consistent research structure. Another distinguishing feature of Galilean geometry is that it demonstrates the beneficial geometric idea of duality. For these reasons, I believe that a mathematics program for teachers' colleges should include a comparative study of three simple geometries, namely Euclidean geometry, the geometry associated with the Galilean principle of relativity, and the geometry associated with Einstein's principle of relativity, as well as an introduction to the specific theory of relativity [1].

Finally, the term "Galilean geometry" is historically inaccurate: Galileo, whose works date from the early 17th century, was unaware of this geometry, whose discovery was inevitably preceded by one of the 19th century's most significant intellectual successes, the creation of the idea that numerous acceptable geometric systems exist. A more precise designation would be "the geometry associated with the Galilean principle of relativity." Because this name is too long to be used repeatedly, we regretfully chose to use the term "Galilean geometry." This moniker is partly justified by Galileo's outstanding clarity and completeness in formulating his theory of relativity, which leads straight to the non-Euclidean geometry discussed in this article [2],[3].

The Galilean [1], [4] plane is conceivable in the affine plane, and the distance between two points was defined as the projection of the points on the x-axis. If the projection of the abscissas is equal to zero, the length is equivalent to the projection of the point on the y-axis. For this, lines parallel to the y-axis are drawn, called special lines in the Galilean plane.

As it is known, the geometric location of the points equidistant from the given point in the Euclidean plane is called the circle. If we look at the geometric location of the points providing this description in the Galilean plane, it will be seen that these are two lines parallel to the y-axis. So, the circle consists of two special straight lines in the Galilean plane.

We can also define the circle in the Euclidean plane as the geometric location of the points seen by the line segment's exact perimeter. If we look at the geometric location of the points in the Galilean plane that satisfy this definition, it will be seen as a parabola whose y-symmetry axis is a special line.

Now let's try to remember these two features in the Euclidean plane that we have read and learned and have been doing lots of exercises on. As you know, a tangent is a line in the plane of a circle that intersects the circle at exactly one point. Tangent lines to circles from the subject of several theorems play an essential role in many geometrical constructions and proofs. Since the tangent line to a circle at any point, A is perpendicular to the radius to that point, theorems involving tangent lines often involve radial lines and orthogonal circles.

2. Methodology

In this section, the basic information will be given about the tangent and secant lines to circles described by Euclidean geometry.

Theorem 2.1 Two congruent right triangles are formed by two tangent segments drawn from the same point outside the circle and have the same length [5].

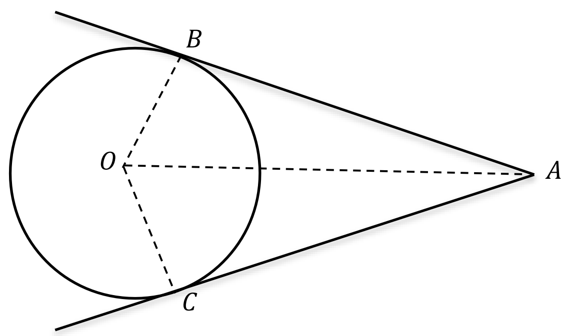

Figure 1: Tangent Segments

This aim is to prove that $AC = AB$ in ΔOCA and ΔOBA. This can be shown briefly and practically as follows:

Step 1. $OC = OB$ (radii)
Step 2. $\angle OCA = \angle OBA = 90^0$ ($OC \perp$ AC, $OB \perp$ AB
Step 3. OA is common to both triangles.
$\Delta OCA \equiv \Delta OBA$ (Right angle, Hypotenuse, Side)
$\therefore AC = AB$

As you know again, a line that intersects a circle at exactly one point is called a tangent, and the point where the intersection occurs is called the point of tangency. The tangent is always perpendicular to the radius drawn to the point of tangency.

Theorem 2.2 Suppose the tangent and secant segments are drawn to a circle from an exterior point. In that case, the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment [6].

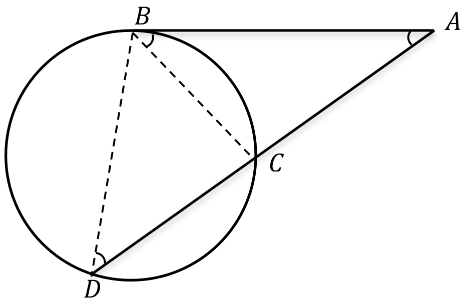

Figure 2: Tangent and Secant segments

Given that AB is a tangent, and ACD is a secant segment.

This aim is to prove that $AB^2 = AC.AD$, and this can be shown briefly and practically as follows:
Step 1. Draw $\overline{BD}$ and $\overline{BC}$ in ΔACB and ΔADB
Step 2. $\angle ABC = \angle ADB$ (Alternate segment angle)
Step 3. $\angle BAC = \angle BAD$ (Common angles)
Step 4. $\therefore \Delta ACB \sim \Delta ADB$ (A-A similarity)

$$\frac{AB}{AD} = \frac{AC}{AB} \Rightarrow AB^2 = AC.AD$$

Hence, it is proven.

We can remember that the proofs of the features we tried to explain briefly above and that we did many exercises on them in our geometry lessons were also shown to us easily and understandably by our teachers.

We can examine the states of these properties, which we learned in the Euclidean plane, in the Galilean plane as follows. Before that, I would like to share more information about the concept of distance in the Galilean plane. After this preliminary, we can dwell on the distance concept in the Galilean plane and continue with the following definitions.

The simplest of non-Euclidean geometry on the plane is Galilean geometry. It is enough for students to grasp the basic concepts of Euclidean geometry and the features of the coordinate system to visualize Galilean geometry. The point, straight line, and parallelism in Galilean geometry are the same as in Euclidean geometry. The only difference between these geometries is the different definitions of the distance between two points.

Let's take the distance between two given points $A(x_1, y_1)$ and $B(x_2, y_2)$ as follows; $d_1 = |x_2 - x_1|$, if $d_1 = 0$ then $d_2 = |y_2 - y_1|$. If $d_1 = 0$ and $d_2 = 0$ then $A = B$. Therefore, it may be concluded that these points coincide [1]. It can be phrased to help students understand it more practically as follows:

$$d = \begin{cases} |x_2 - x_1| , \ x_2 \neq x_1; \\ |y_2 - y_1|, \ x_2 = x_1. \end{cases}$$

This distance in the Galilean plane G^2 has its meaning in the Euclidean plane E^2. The projection of the cross-section connecting the two points on the Ox axis. If the projection on the Ox axis is a point, then the projection on the Oy axis is obtained.

Definition 2.1 The geometric location of the points seen by the unchanging perimeter of the non-special line segment given in the geometric plane is called the cycle [7]. We can consider the cycle in the Galilean plane to confirm the definition of the circle in Euclidean geometry.

If the xOy coordinate system is given in the Galilean plane, the cycle equation is written as $y = ax^2 + bx + c$ [8].

Using the parallel shift method of the coordinate axes, we can write the cycle equation in a simpler view. In a truthful way,

$$y = a(x^2 + 2\frac{b}{2a}x + \frac{b^2}{4a^2}) + c - \frac{b^2}{4a} \Rightarrow y = a(x + \frac{b}{2a})^2 - \frac{b^2 - 4ac}{4a}$$

$$\Rightarrow y + \frac{b^2 - 4ac}{4a} = a(x + \frac{b}{2a})^2$$

can be obtained. Here,

$$\begin{cases} X = x + \dfrac{b}{2a} \\ Y = y + \dfrac{b^2 - 4ac}{4a} \end{cases}$$

shifting the coordinates head gives the equation for the simple form of the cycle $y = ax^2$. The cycle's axis of symmetry is $x = \frac{b}{2a}$ a special line. As is commonly understood, motion in the Galilean plane [4],

$$\begin{cases} X = x + x_0 \\ Y = hx + y + y_0 \end{cases}$$

it consists of changing the lines of the y -parallel and is formed by turning it to the h angle. Now let's try to prove these properties of the cycle.

3. Properties

Property 3.1 a is a constant variable with a cycle coefficient. Here the coefficient a is also called the radius of the cycle [1].

If $a = 0$ so, the cycle would be an ordinate axis. If the radius $a > 0$ of the cycle is in the direction of the ordinate axis, if $a < 0$, the cycle is in the opposite direction to the ordinate axis.

Of course, the cycle divides the Galilean plane into two regions. If $a > 0$, the region located in the positive direction of the plane axis is called the inner region of the cycle, the second part of the plane is called the outer region of the cycle.

Property 3.2 Any two tangent lines can be drawn from the point $M(x_0, y_0)$ that is outside the cycle.

For this, let's take the cycle $y = ax^2 + bx + c$ and the point $M(x_0, y_0)$ outside the cycle. The line equation which passes through the point $M(x_0, y_0)$ is $y - y_0 = k(x - x_0)$.

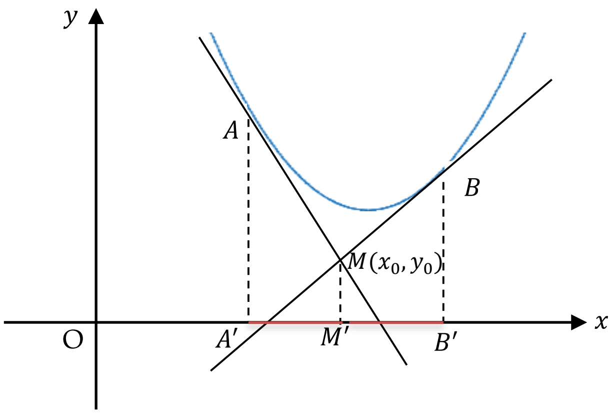

Figure 3: Cycle with tangent lines

$$M'A' = M'B'$$

Intersecting points of this line with the cycle

$$\begin{cases} y = ax^2 + bx + c \\ y = k(x - x_0) + y_0 \end{cases}$$

is the solution of the system given above. From here, for the abscissa of the intersection points,

$$x_{1,2} = \frac{(k-b) \pm \sqrt{(k-b)^2 - 4a(c + kx_0 - y_0)}}{2a} \quad (1)$$

we can form the result. For the line to be tangent to the cycle, the intersection points must overlap. That is, the system must have a solution.

From here, to be $x_1 = x_2$,

$$(k - b)^2 - 4a(c + kx_0 - y_0) = 0 \text{ or } k^2 - 2(b + 2a_0)k + b^2 - 4a(c + 4a)y_0 = 0$$

should be.

Thus the coefficient of tangent angle;

(2) $$k_{1,2} = (b - 2ax_0) \pm \sqrt{4a(ax_0^2 + bx_0 + c - y_0)}$$

If $a > 0$ and $ax^2 + bx_0 + c - y_0 > 0$, then the equation would have two different $k_1 \neq k_2$ solutions. Because the given point $M(x_0, y_0)$ is outside the cycle, this requirement was completed. Hence, the second property was proved too.

Property 3.3 The tangent lines can be drawn to a cycle from a point that is outside of the cycle. Thus the lengths of the segments from a point that is outside of the cycle to the two tangent points are equal. Let's clarify the tangency points of the tangents drawn from the given point $M(x_0, y_0)$ to the cycle. For this, by adding the values of k_1 and k_2 in (1) to the equality (2) if we determine the abscissa,

$$x_{1,2} = x_0 \pm \sqrt{\frac{1}{a}(ax_0^2 + bx_0 + c - y_0)}$$

obtained.

The distance from the point $M(x_0, y_0)$ to the distance of tangency points,

(3) $$|x_1 - x_0| = |x_2 - x_0| = \sqrt{\frac{1}{a}(ax_0^2 + bx_0 + c - y_0)}$$

exists with this equation, and this equation shows the proof of property.

Property 3.4 Let the point L be given outside the cycle, and LA line that intersects the point A and B is drawn. For the given cycle $LA.LB$ depends on only the point L.

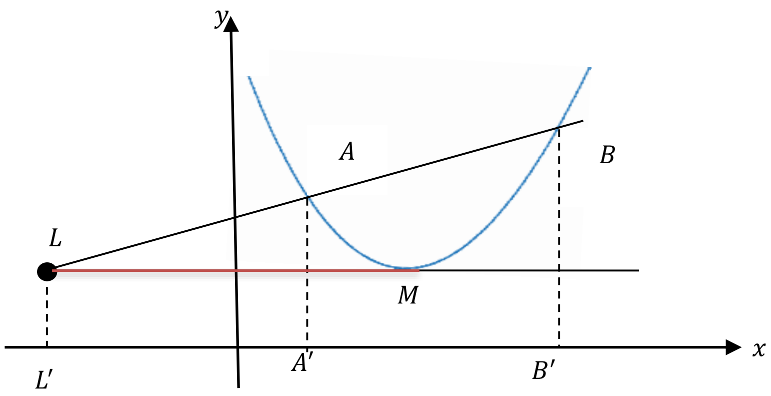

Figure 4: Cycle with tangent and secant lines

$$LA.LB = LM^2$$

If we show the coordinates of point L by (x_0, y_0), and the coordinates of the point A and B by (x_1, y_1), (x_2, y_2) then x_1, x_2 is confirmed by the equation (1).

From here, since $LA = |x_0 - x_1|$, and $LB = |x_0 - x_2|$,

(4) $$LA.LB = |x_0^2 - x_0(x_1 + x_2) + x_1.x_2| = \left|x_0^2 - x_0\frac{b-k}{a} + \frac{kx_0 - y_0 + c}{a}\right|$$

$$= \frac{1}{a}\left|ax_0^2 + bx_0 + c - y_0\right| = LM^2$$

derived from property 3.3. So, the $LA.LB$ product depends only on the coordinates of the L point and the cycle parameters, proving that the property is correct.

Theorem 3.1 Let's draw a line from a given L point that intersects the cycle at A and B points. For those lines that are secant and tangent; If a tangent and a secant are drawn from an external point to a cycle, then the square of the length of the tangent LC is equal to the product of the length of the secant's external part LA and the length of the entire secant LB.

The proof of the theorem is derived from equations (4) and (3). The properties and theorem we tried to explain above show that the cycle in the Galilean plane has the properties of the circle in the Euclidean plane.

4. Recommendations

If educators are obliged to teach non-Euclidean geometry, they must first study the material or have previously reviewed it. According to the above principles, teachers who study non-Euclidean geometry will be mathematically more competent than those who do not. Even if non-Euclidean geometry is not part of the school curriculum, teachers must be conversant with it to effectively teach Euclidean geometry. Non-Euclidean geometry, however, is only taught in some colleges and universities' mathematics programs for future educators.

As a result, in-service courses must be developed to allow teachers to study topics in mathematics, such as non-Euclidean geometry, which are on the outskirts of the conventional school curriculum yet profoundly linked to it. This will enable teachers to teach with greater confidence, which is required if they are to rely on their knowledge and skills rather than coercive tactics to ensure that pupils learn properly [9].

Attempts by some of the most skilled mathematicians over the last two thousand years to establish the parallel postulate or construct an appropriate substitute postulate have failed miserably. Because this disagreeable truth could not be explained, the parallel postulate could not be demonstrated using other definitions, shared ideas, or postulates. As a result, a study focused on the geometries that followed the parallel postulate's denial. The discovery of non-Euclidean geometry was thus a foregone conclusion.

Applying appropriate teaching methodologies is crucial to realizing the characteristics that first inspired secondary school students to explore non-Euclidean geometry. Students must first exhibit the skills associated with the rank of formal deduction before studying non-Euclidean geometry in depth since geometric reasoning develops in phases.

For example, in an axiomatic system such as Euclidean geometry, students should understand the concepts of terms, definitions, postulate, theorems, and proof and their interrelationships. This ability can be tested by assigning students a test item in which they must classify claims as definitions, postulates, or theorems, and then determine which can be inferred from the ones they have classified as theorems.

The discovery technique is the most appropriate teaching technique because the new topic matter mainly involves new concepts and principles. However, there are drawbacks to utilizing the discovery technique, such as children being upset and discouraged if they fail to discover continuously. This problem can be solved in several ways.

The foundations of Euclidean geometry, for example, can be appropriately introduced through a teacher-led or group-based discussion. In contrast, alternative geometry can be better introduced by having students interact with manipulative objects and remark on their discoveries. Students will build new knowledge based on past information and experiences and reorganize it to fit the new knowledge. Unfortunately, teachers frequently concentrate on teaching strategies that only target the lowest levels of cognition because most school-level mathematics assessments entail memorizing facts and procedures. Since students cannot acquire these qualities in the classroom, they cannot be held accountable for their lack of originality, independence, or creativity. Discussions, oral and written presentations, quizzes, and research should all be encouraged by teachers [9]. These could be helpful methods for routinely evaluating pupils' understanding of new content.

5. Conclusion

We should take full advantage of non-Euclidean geometry's capacity to give our students fresh insights into the world and their place as math educators. The potential for non-Euclidean geometry to be a significant part of the mathematics curriculum has been shown. False assumptions about the beginning and nature of mathematics can be disproved, and workable alternatives can be offered by studying non-Euclidean geometry.

Non-Euclidean geometry is a more accurate representation of our world than Euclidean geometry claims to be. It is important to emphasize that non-Euclidean geometry may be recommended for all appropriate students.

When the specific conclusions gained in this study are compared to the goals described above, it is evident that learning non-Euclidean geometry can assist in achieving them, meaning that non-Euclidean geometry can play an essential role in the syllabus.

The students will enjoy these exciting topics. They will admire it and be fascinated, we should manage to get their attention, and finally, they will be so interested in non-Euclidean geometry that we will be astounded.

To summarize, the cycle in the Galilean plane can be viewed as a circle in the Euclidean plane. Is the Galilean cycle identical to the Euclidian cycle? This concern was mentioned in response to the previous inquiry. I hope the cycle and properties I discovered as an innovator and sought to present to you will occur. They deserve to be included in the mathematics curriculum at the most reasonable time in this study, from which I have profited with valuable resources.

6. Author’s Contribution

We confirm that the manuscript has been read and approved by all named authors. We also confirm that each author has the same contribution to the paper. We further confirm that the order of authors listed in the manuscript has been approved by all authors.

7. Conflict of interest

There is no conflict of interest for this paper.

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