Eurasian J. Sci. Eng., 2023, 9(2), 54-64
https://doi.org/10.23918/eajse.v9i2p5
Published by Tishk International University
Available at: https://eajse.tiu.edu.iq/

RESEARCH ARTICLE Open Access

APPLICABILITY OF THE LAMINATED PLATE THEORIES ON REINFORCED CONCRETE SLAB

Sameer Mohammedsaleem Ahmed[1*] and Salahuddin Abdulrahman Ahmed[1]

[1*]Department of Civil engineering, Faculty of Engineering, Salahaddin University, Erbil-IRAQ

Article History
Received: 14.10.2022
Revised: 01.04.2023
Accepted: 29.04.2023
Communicated by: Dr. Orhan Tug
*Email address:
Samir.barzaniii96@gmail.com
Corresponding Author

Abstract: The purpose of this study is to indicate the applicability of laminated plate theory to reinforced concrete (RC) slabs in the elastic stage. For static bending analysis of reinforced concrete (RC) slabs, analytical methods are applied, such as the first order shear deformation theory (FSDT) and the classical laminated plate theory (CLPT). This research takes into account that an RC slab is made up of several layers of concrete and steel bars that are bonded together. Two cases of simply supported square slabs, in which the slab thickness varies, have been considered. The deflection and stresses on the center of the RC slab have been calculated by using Navier approaches of the classical laminated plate theory of plates and the first order shear theory. The results were compared to the experimental and FEM findings of the previous study. The present research concluded that the results of the deflection predicted by laminated plate theory coincide with the experimental results with sufficient accuracy, and CLPT fails to take into account shear strains at the interfaces that making it less applicable compared to the FSDT. However, the application of the laminated plate theory formulas to determine the stresses for RC slab gave a bit different in the results compared with experimental results from a previous study. Also, the results of laminated plate theories are heavily influenced by a variety of assumptions.

Keywords: *CLPT; FSDT; RC Slab; Deflection; Stress.*

1. Introduction

Reinforced concrete is a composite material, this indicates that it is made up of various constituent materials with a wide range of properties that work well together. In the case of reinforced concrete, concrete and steel are most commonly used as component materials [1]. A flat composite structural element is typically thought of as a laminated plate, which is made up of multiple layers bonded together, each with a different strength. Stiffness, weight reduction, and other characteristics that could be enhanced by making a composite [2]. A long time ago, the effectiveness of classical methods and first order shear theory in the analysis of laminated plates was proven. Some technical aspects must be considered when analyzing the structural behavior of composite laminated structures, such as reinforced concrete slabs. Going back to the history of plates, Euler made the first inference to the plate issues mathematical statement in 1776 [3]. Since then, many differential equations have been invented. In the 19th century, many plate theories have been developed, including Navier's, Bernoulli's, Lagrange's, Saint- Venant's, and Kirchhoff's, which introduced theories for thin plate analysis [4]. As an extension of the classical plate theory (CPT), the classical laminated plate theory (CLPT), applied to laminated plates, was the first theory for analyzing laminated plates [5]. Reddy and Robbins Jr [6] exhibited a study of multiple layer-wise laminated plate theories and equivalent single layer. Liu and Li [7] developed a summary of laminated plate theories according to generalized Zigzag theories, the suggested global-local double-superposition theories, layer-wise theories, and the displacement theories. Altenbach [8] shown off a review of theories for sandwich plates and laminated. Ghugal and Shimpi [9] discussed different equivalent layer-wise and single layer laminated plate theories based on displacement and stress. Reddy and Arciniega [10] presented a summary of shell theories and shear deformation plate. Kant and Swaminathan [11] provided a selective study and overview of theories, with a focus on transverse/interlaminar stress estimate in laminated composites, since 1967, Mittelstedt and Becker [12] conducted a selective literature review on the free-edge impact. Pagano applied the elasticity theory of three-dimensional to provide analytical solutions for laminated plates in bending, he demonstrated that when the side to ratio of thickness increases, the classical laminated plate theory (CLPT) becomes less accurate [13]. Shvan [14] tested eight concrete slabs that the results indicated that the classical plate theory formulas to determine the stresses within elastic range gave a significant error in the result compared with the experimental, while the value of the deflection agreed with the experimental. Classic theories have been less applicable as computers have developed and new software based on the FEM has become available [15]. This study is aimed to apply First order shear theory and Classical laminated plate theory to evaluate maximum deflection and bending stresses values in square reinforced concrete slabs, laminated plate theory analytical findings were compared to experimental and FEM results from previous studies.

2. Methodology

2.1 Classical Laminated Plate Theory

The classical laminated plate theory (CLPT) is a composite laminates extension of the classical plate theory (CPT), although it is unsuitable for flexural analysis of moderately thick laminates [16]. According to Reissner and Stavsky [5], it provides relatively accurate answers to a variety of engineering issues, including thin composite plates. The transverse shear stress components are ignored in this theory, which models a laminate as a layer of equivalent singles.

In Kirchhoff theory, straight lines normal to the xy-plane before deformation stay straight and normal to the mid-surface after distortion, according to a fundamental assumption for plate bending deformation [2].

Based on this assumption, the CLPT displacement field is as below:

(1) $$c(x,y,z) = c_o(x,y) - z\frac{\partial \omega_o}{\partial x}$$

(2) $$d(x,y,z) = d_o(x,y) - z\frac{\partial \omega_o}{\partial y}$$

(3) $$\omega(x,y,z) = \omega_o(x,y)$$

where c_o, d_o and ω_o are the displacements with the coordinate lines of a xy-plane material point. To calculate the strains, utilize the following strain-displacement relations for small strains:

(4) $$\epsilon_{xx} = \frac{\partial c_o}{\partial x} - z\frac{\partial^2 \omega_o}{\partial x^2}$$

(5) $$\epsilon_{yy} = \frac{\partial d_o}{\partial y} - z\frac{\partial^2 \omega_o}{\partial y^2}$$

(6) $$\epsilon_{xy} = \frac{1}{2}\left(\frac{\partial c_o}{\partial y} + \frac{\partial d_o}{\partial x}\right) - z\frac{\partial^2 \omega_o}{\partial x \partial y}$$

In CLPT, the transverse strains are zero.

As a result, the constitutive equations are as follows:

(7)
$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{62} & A_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx}{}^{o} \\ \epsilon_{yy}{}^{o} \\ \epsilon_{xy}{}^{o} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{61} & B_{62} & B_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx}{}^{1} \\ \epsilon_{yy}{}^{1} \\ \epsilon_{xy}{}^{1} \end{Bmatrix}$$

(8)
$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{61} & B_{62} & B_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx}{}^{o} \\ \epsilon_{yy}{}^{o} \\ \epsilon_{xy}{}^{o} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{21} & D_{22} & D_{26} \\ D_{61} & D_{62} & D_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx}{}^{1} \\ \epsilon_{yy}{}^{1} \\ \epsilon_{xy}{}^{1} \end{Bmatrix}$$

Where B_{ij} the bending-extensional coupling stiffnesses, D_{ij} the bending stiffnesses and A_{ij} are extensional stiffnesses.

As a result, the governing equations are as follows:

(9)
$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0$$

(10)
$$\frac{\partial N_{yy}}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0$$

(11)
$$\frac{\partial^2 M_{xx}}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial y \partial x} + \frac{\partial^2 M_{yy}}{\partial y^2} = q$$

2.2 First Order Shear Deformation Theory

The transverse planes that are initially normal and parallel to the plate's mid-plane, are supposed to stay straight but not inevitably normal after deformation in the first order shear deformation theory (FSDT), and for modifying the transverse shear stress, shear correction factors are applied, and all across the thickness of the plate are constant [2].
Solutions based on FSDT; the below displacement field could be supposed:

(12)
$$c(x, y, z) = c_o(x, y) - z\phi_x$$

(13)
$$d(x, y, z) = d_o(x, y) - z\phi_y$$

(14)
$$\omega(x, y, z) = \omega_o(x, y)$$

Where ϕ_y and ϕ_x are the rotations of a transverse normal about the x- and y-axes, respectively. The strain-displacement relations for small strains in FSDT are:

(15)
$$\epsilon_{xx} = \frac{\partial c_o}{\partial x} + z\frac{\partial \phi_x}{\partial x}$$

(16)
$$\epsilon_{yy} = \frac{\partial d_o}{\partial y} + z\frac{\partial \phi_y}{\partial y}$$

(17)
$$\epsilon_{xy} = \frac{1}{2}\left(\frac{\partial c_o}{\partial y} + \frac{\partial d_o}{\partial x}\right) + z\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right)$$

As a result, the constitutive equations are as follows:

(18) $$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{62} & A_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx}{}^{o} \\ \epsilon_{yy}{}^{o} \\ \epsilon_{xy}{}^{o} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{61} & B_{62} & B_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx}{}^{1} \\ \epsilon_{yy}{}^{1} \\ \epsilon_{xy}{}^{1} \end{Bmatrix}$$

(19) $$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{61} & B_{62} & B_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx}{}^{o} \\ \epsilon_{yy}{}^{o} \\ \epsilon_{xy}{}^{o} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{21} & D_{22} & D_{26} \\ D_{61} & D_{62} & D_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx}{}^{1} \\ \epsilon_{yy}{}^{1} \\ \epsilon_{xy}{}^{1} \end{Bmatrix}$$

K is the shear correction coefficient and its value is 5/6 [2].

As a result, the governing equations are as follows:

(20) $$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0$$

(21) $$\frac{\partial N_{yy}}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0$$

(22) $$\frac{\partial Q_{x}}{\partial x} + \frac{\partial Q_{y}}{\partial y} = q$$

(23) $$\frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = 0$$

(24) $$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{yy}}{\partial y} - Q_y = 0$$

2.3 Navier Approaches

For rectangular laminates simply supported with stiffnesses of A_{16}, A_{26}, B_{16}, B_{26}, D_{l6}, D_{26}, and A_{45} that are zero, Navier solutions can be applied [2].

boundary conditions:

$c_o(x,0) = 0,\ c_o(x,b) = 0,\ d_o(0,y) = 0,\ d_o(a,y) = 0, \omega_o(0,y) = 0,\ \omega_o(a,y) = 0,$

$\omega_o(x,0) = 0, \omega_o(x,b) = 0,\ N_{xx}(0,y) = 0,\ N_{xx}(a,y) = 0,\ N_{yy}(x,0) = 0,$

$N_{yy}(x,b) = 0, M_{xx}(0,y) = 0,\ M_{xx}(a,y) = 0,\ M_{yy}(x,0) = 0, M_{yy}(x,b) = 0,$

$\phi_y(0,y) = 0, \phi_y(a,y) = 0,\ \phi_x(x,0) = 0\ ,\ \phi_x(x,b) = 0\ .$

The following expansions satisfy the boundary conditions:

(25) $$c_o(x,y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{mn} \cos\alpha x \sin\beta y$$

(26) $$d_o(x,y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D_{mn} \cos\beta y \sin\alpha x$$

(27) $$\omega_o(x,y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} W_{mn} \sin\alpha x \sin\beta y$$

(28) $$\phi_x(x,y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} X_{mn} \cos\alpha x \sin\beta y$$

(29) $$\phi_y(x,y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} Y_{mn} \cos \beta y \sin \alpha x$$

Where $\alpha = \frac{m\pi}{a}$ / a and $\beta = \frac{n\pi}{b}$, and C_{mn} , D_{mn}, W_{mn}, X_{mn} , Y_{mn} are coefficients to be determined.

In double sine series, the mechanical transverse load q must be extended. Therefore:

(30) $$q(x,y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} Q_{mn} \sin \alpha x \sin \beta y$$

For distributed load:

(31) $$Q_{mn} = \frac{4\,q}{mn\pi^2}(\sin \alpha x \sin \beta y)$$

differential operator form can be written in CLPT by expanding loads and generalized displacements in a double trigonometric series according to unknown parameters:

(32) $$\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{Bmatrix} c_o \\ d_o \\ \omega_o \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ Q_{mn} \end{Bmatrix}$$

where coefficients k_{ij} are defined by:

$k_{11} = A_{11}\alpha^2 + A_{66}\beta^2$, $k_{12} = (A_{12} + A_{66})\alpha\beta$, $k_{13} = -(3B_{16}\alpha^2 + B_{26}\beta^2)\beta$,

$k_{22} = A_{66}\alpha^2 + A_{22}\beta^2$, $k_{23} = -(B_{16}\alpha^2 + 3B_{26}\beta^2)\alpha$, $k_{33} = D_{11}\alpha^4 + (2D_{12} + D_{66}\beta^2)\alpha^2\beta^2 + D_{22}\beta^4$.

In FSDT, for the identical laminates used in traditional laminate theory. The coefficients (C_{mn}, D_{mn}, W_{mn}, X_{mn}, Y_{mn}) of the Navier solution for these laminates could be calculated as:

(33) $$\begin{bmatrix} m_{11} & m_{12} & 0 & m_{14} & m_{15} \\ m_{21} & m_{22} & 0 & m_{24} & m_{25} \\ 0 & 0 & m_{33} & m_{34} & m_{35} \\ m_{14} & m_{24} & m_{34} & m_{44} & m_{45} \\ m_{15} & m_{25} & m_{35} & m_{45} & m_{55} \end{bmatrix} \begin{Bmatrix} C_{mn} \\ D_{mn} \\ W_{mn} \\ X_{mn} \\ Y_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ Q_{mn} \\ 0 \\ 0 \end{Bmatrix}$$

Where coefficients m_{ij} are defined by:

$m_{11} = A_{11}\alpha^2 + A_{66}\beta^2$, $m_{12} = (A_{12} + A_{66})\alpha\beta$, $m_{14} = B_{11}\alpha^2 + B_{66}\beta^2$,

$m_{15} = (B_{12} + B_{66})\alpha\beta$, $m_{22} = A_{66}\alpha^2 + A_{22}\beta^2$, $m_{24} = m_{15}$, $m_{25} = B_{66}\alpha^2 + B_{22}\beta^2$,

$m_{33} = K(A_{55}\alpha^2 + A_{44}\beta^2)$, $m_{34} = KA_{55}\alpha$, $m_{35} = KA_{44}\beta$

$m_{44} = D_{11}\alpha^2 + D_{66}\beta^2 + KA_{55}$, $m_{45} = (D_{12} + D_{66})\alpha\beta$, $m_{55} = D_{66}\alpha^2 + D_{22}\beta^2 + KA_{44}$

2.4 Illustrative Example

As illustrated in Figure 1, a and b are the horizontal and vertical dimensions of the whole plate, and x and y are the coordinates of the location at which the transverse deflection w is calculated.

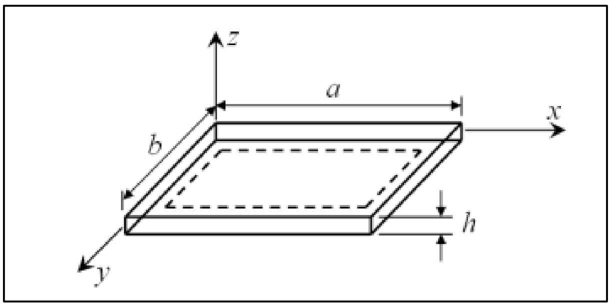

Figure 1: Geometry of the simply supported rectangular laminated plate

The following values are assumed in elastic stage:

Plate dimensions, a =950 mm & b=950 mm.

Poisson's reinforcement ratio =0.3.

Young's reinforcement modulus in all direction =200 GPa.

Shear modulus of reinforcement in all direction =76.92 GPa

Clear cover of concrete= 8 mm

Steel reinforcement: Ø10@100mm both directions with equivalent thickness of 2 mm as a steel plate for 90 mm thickness and Ø10@150mm both directions with equivalent thickness of 1.26 mm as a steel plate for 60 mm thickness.

Poisson's concrete ratio = 0.16.

Young's modulus of concrete in all direction =35.15 GPa.

Shear modulus of concrete in all direction =15.15 GPa.

Applied partially distributed load over area (95 mm×95 mm) on the center =1.66 MPa.

Cross section of RC slabs is illustrated in Figures 2 and 3.

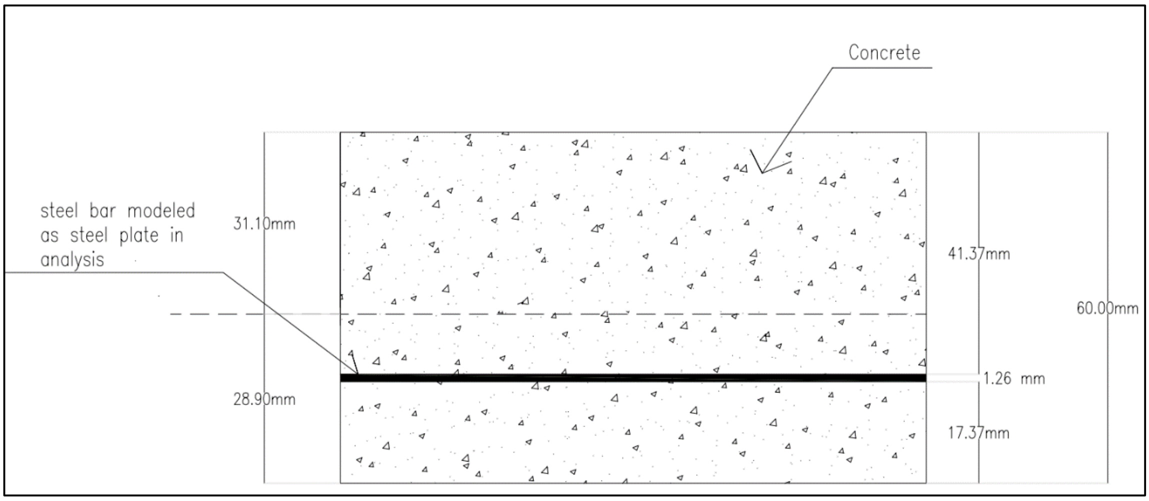

Figure 2: Cross section of RC slab with thickness of 60 mm.

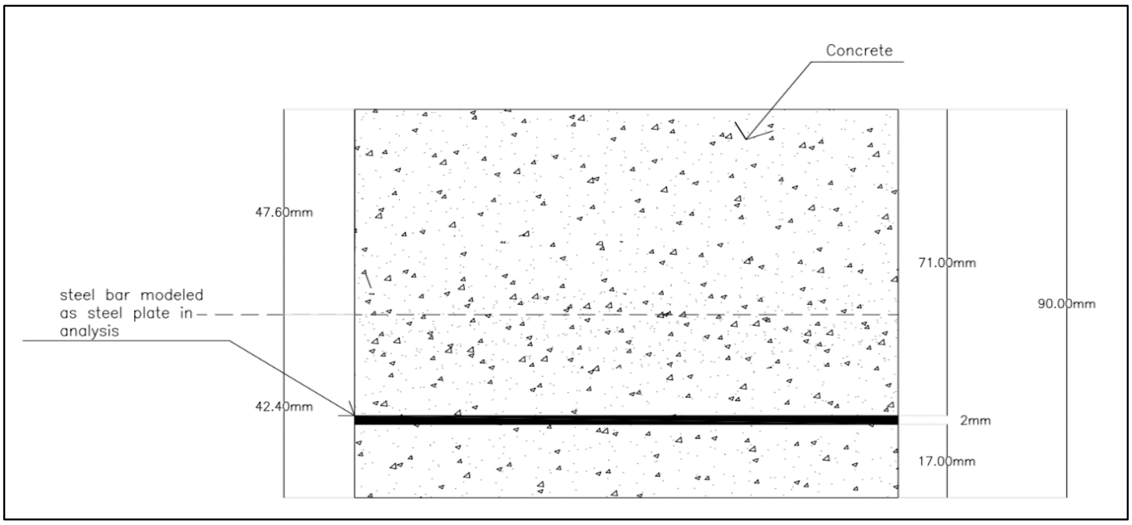

Figure 3: Cross section of RC slab with thickness of 90 mm.

3. Results and Discussion

Deflections of square RC slabs with two thickness groups that are simply supported (60 mm and 90 mm) under its self-weight and partially distributed load were 0.208 according to classical laminate plate theory and 0.209 according to first order shear theory for slabs with a thickness of 60 mm, and were 0.0575 according to classical laminate plate theory and 0.0601 according to first order shear theory for slabs with a 90 mm thickness. Experimental and FEM values are also presented according to the Shvan [14] from previous study, as illustrated in Table 1.

Table 1: Deflections of simply supported square RC slab.

Thickness (mm)	CLPT (mm)	FSDT (mm)	Experimental(mm) [14]	FEM (mm) [14]
60	0.205	0.209	0.21	0.199
90	0.0575	0.0601	0.07	0.072

According to the findings of this study, a square laminated plate's highest deflection occurs near the slab's center, the highest deflection (w_z) obtained by classical laminate plate theory and experimental results from previous study were compared. The results were close to 97.62 % and 82.14 % for square slabs with thicknesses of 60 mm and 90 mm, respectively. The maximum deflection (w_z) obtained by FSDT and experimental results from previous study were compared, the results were close to 99.5 % and 85.86 % for square slabs with thicknesses of 60 mm and 90 mm, respectively. According to Shvan [14] the maximum deflection (w_z) achieved through FEM and experimental were compared, the findings were close to 94.76 % and 97.22 % for square slabs with thicknesses of 60 mm and 90 mm, respectively. The results of maximum deflection were shown graphically for RC slab with thicknesses of 60 mm as shown in Figure 4.

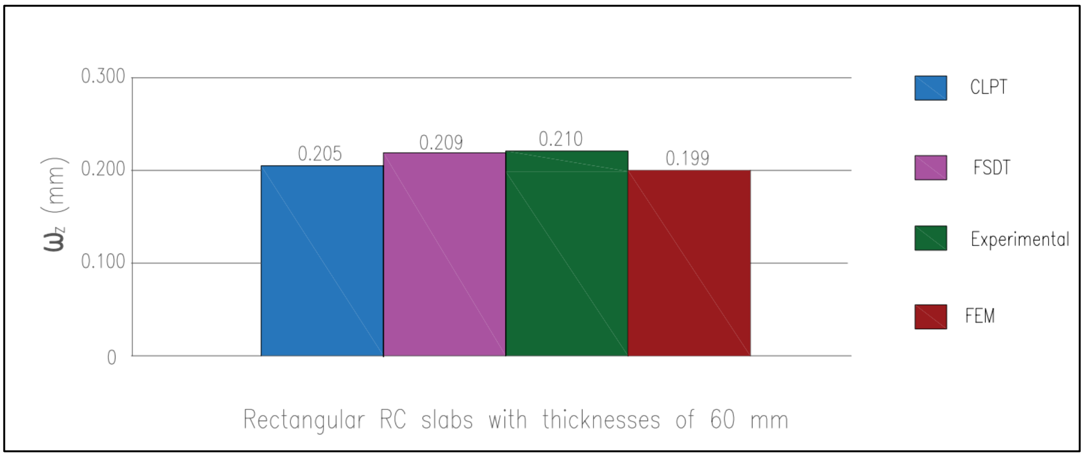

Figure 4: the maximum deflection which obtained by different methods for RC slab with thicknesses of 60 mm.

The results of maximum deflection were shown graphically for RC slab with thicknesses of 90 mm as shown in Figure 5.

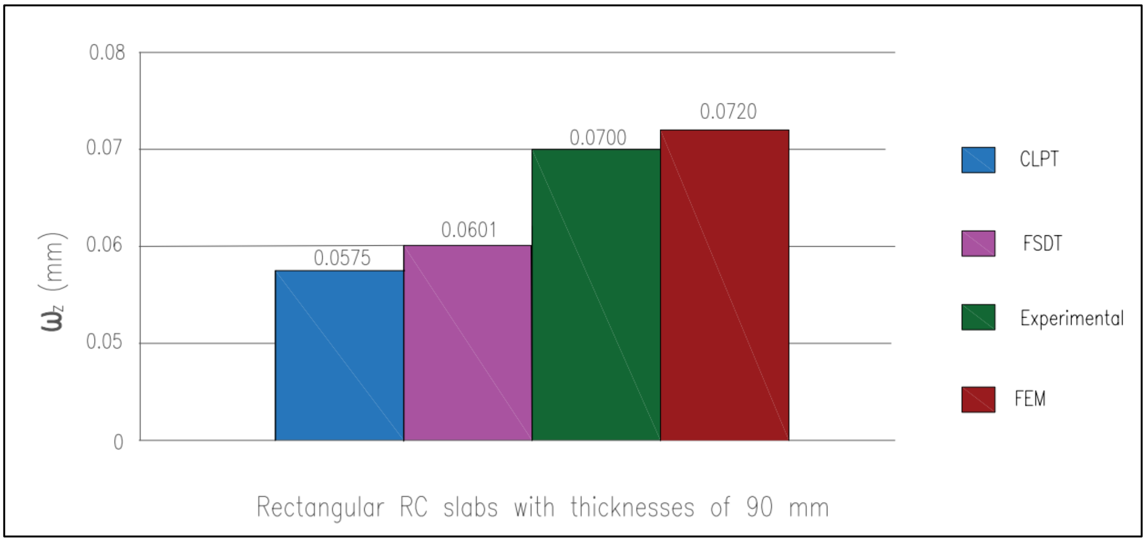

Figure 5: The maximum deflection which obtained by different methods for RC slab with thickness of 90 mm.

Stresses of simply supported square slabs with two groups of thickness that are (90 mm and 60 mm) under self-weight and partially distributed load have been predicted and estimated according first order shear theory and classical laminate plate theory were 2.96 for slabs with thicknesses of 60 mm, while were 1.33 for slabs with thicknesses of 90 mm, experimental and FEM values are also presented according to the Shvan [14] from previous study, as illustrated in Table 2.

Table 2: Stresses of simply supported square RC slab.

Thickness (mm)	CLPT (MPa)	FSDT (MPa)	Experimental (MPa) [14]	FEM (MPa) [14]
60	2.96	2.96	3.34	3.851
90	1.33	1.33	1.78	1.885

The current investigation determined that the maximum stress of a square laminated plate is located in the center of the slab, the maximum stress (σ_{xx}) obtained by classical laminate plate theory and experimental results from previous study were compared. The results were close to 88.62 % and 74.72 % for square slabs with thicknesses of 60 mm and 90 mm, respectively, the maximum stress (σ_{xx}) obtained by FSDT is the same as CPLT. According to the Shvan [14] that the maximum stress (σ_{xx}) was obtained by FEM and experimental were compared, the results were close to 86.73 % and 94.4 % for square slabs with thicknesses of 60 mm and 90 mm, respectively.

The results of maximum stress were shown graphically for RC slab with thicknesses of 60 mm as shown in Figure 6.

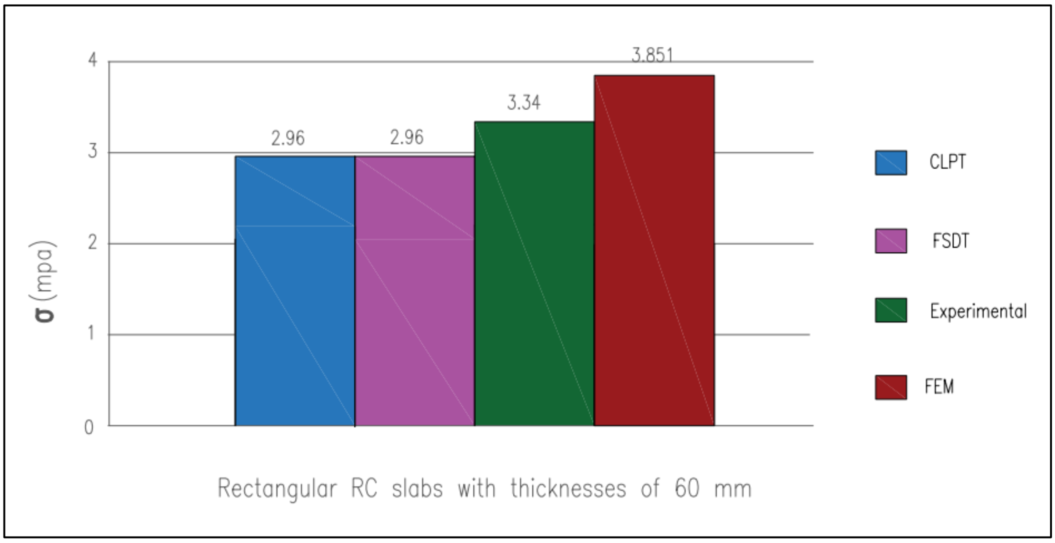

Figure 6: The maximum stress which obtained by different methods for RC slab with thicknesses of 60 mm

The results of maximum stress were shown graphically for RC slab with thicknesses of 90 mm as illustrated in Figure 7.

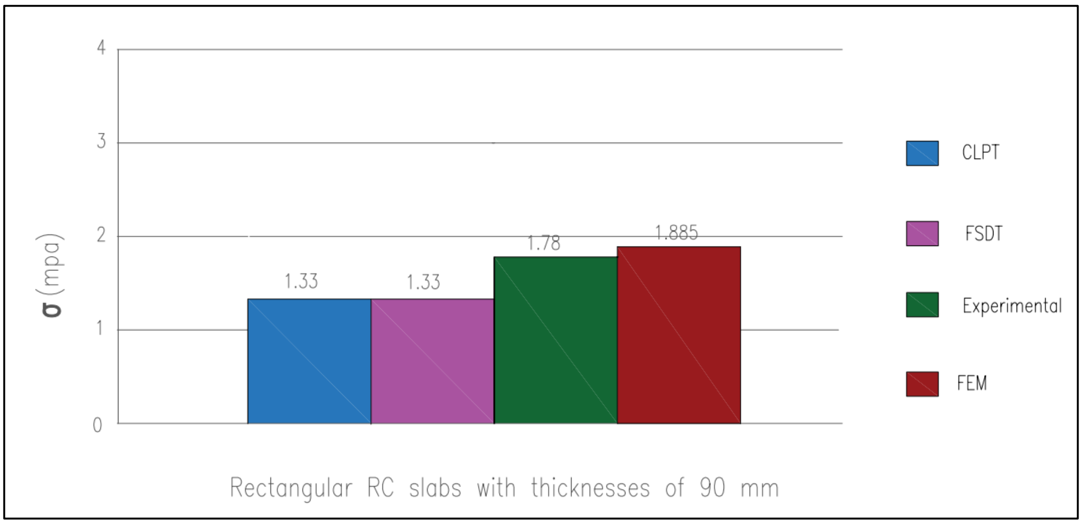

Figure 7: The maximum stress which obtained by different methods for RC slab with thicknesses of 90 mm

The low percentage of differences of deflection results between CLPT and FSDT can be attributed to the fact that the CLPT fails to take into account shear strains at the interfaces. The bending stresses results of the first order shear theory and classical laminated plate show a bit different compared to the experimental results but that there is not an important distinction difference between laminated plate theory and experimental results, and the low difference percentage of the laminated plate theories compared to the fact that CLPT and FSDT are based on assumptions might explain the experimental outcomes, it has been observed that the thickness of slabs has a great effect on the approach to the experimental results of deflection and stresses. The FEA findings were more accurate because they represented an actual model of a concrete square slab with reinforcements, a 3D modelled slab with all of the precise material characteristics [15].

4. Conclusion

The goal of this research was to explore the applicability of the laminated plate theory on a square RC slab under partial uniform distributed load at the center and its self-weight in an elastic stage in order to better understand its behavior. The following conclusions can be drawn:

1. This study confirmed that the results of stresses differed when comparing laminated plate theory and experimental data, but that the differences were acceptable with appropriate accuracy.
2. The results of deflection by CLPT and FSDT agreed with the experimental results, the FSDT that takes into account shear strains at the interfaces is closer to the experimental results.
3. For both analytical approaches, the prediction was more accurate for slabs 60 mm thick rather than 90 mm.
4. Generally, the laminated plate theories were applicable on square RC slab's behavior for displacement and stress at the center in elastic stage.

5. Author's Contribution

Conceptualization, Methodology, Modeling. **Sameer**: Supervision. **Salahuddin.**

6. Conflict of Interest

There is no conflict of interest for this paper.

7. Acknowledgment

The authors of this study would like to thank the head of Salahaddin University-Civil Engineering Department for their assistance.

References

[1] Darwin D, Dolan CW, Nilson AH. Design of concrete structures. New York, NY, USA:: McGraw-Hill Education; 2016.

[2] Reddy JN. Mechanics of laminated composite plates and shells: theory and analysis. CRC press; 2003 Nov 24.

[3] Euler L. De motu vibratorio tympanorum. Novi commentarii academiae scientiarum Petropolitanae. 1766: 243-60.

[4] Ventsel E, Krauthammer T, Carrera EJ. Thin plates and shells: theory, analysis, and applications. Appl. Mech. Rev.. 2002 Jul 1; 55(4): B72-3. https://doi.org/10.1115/1.1483356

[5] Reissner E, Stavsky Y. Bending and stretching of certain types of heterogeneous aeolotropic elastic plates. 1961: 402-408. https://doi.org/10.1115/1.3641719

[6] Reddy JN, Robbins Jr DH. Theories and computational models for composite laminates. 1994. https://doi.org/10.1115/1.3111076

[7] Liu D, Li X. An overall view of laminate theories based on displacement hypothesis. Journal of composite materials. 1996 Sep; 30(14): 1539-61. http://dx.doi.org/10.1007/s11831-019-09392-2

[8] Altenbach H. Theories for laminated and sandwich plates: A review. Mechanics of composite materials. 1998 May; 34(3): 243-52. https://doi.org/10.1007/BF02256043

[9] Ghugal YM, Shimpi RP. A review of refined shear deformation theories for isotropic and anisotropic laminated beams. Journal of reinforced plastics and composites. 2001 Feb; 20(3): 255-72. https://doi.org/10.1177/073168401772678283

[10] Reddy JN, Arciniega RA. Shear deformation plate and shell theories: from Stavsky to present. Mechanics of Advanced Materials and Structures. 2004 Nov 1; 11(6): 535-82.

[11] Kant TA, Swaminathan K. Estimation of transverse/interlaminar stresses in laminated composites–a selective review and survey of current developments. Composite structures. 2000 May 1; 49(1): 65-75. http://dx.doi.org/10.1016/S0263-8223(99)00126-9

[12] Mittelstedt C, Becker W. Interlaminar stress concentrations in layered structures: Part I-A selective literature survey on the free-edge effect since 1967. Journal of Composite Materials. 2004 Jun; 38(12): 1037-62. https://doi.org/10.1177/0021998304040566

[13] Reddy JN. A penalty plate-bending element for the analysis of laminated anisotropic composite plates. International Journal for numerical methods in Engineering. 1980 Aug; 15(8): 1187-206.

[14] Sharif, S.M. Plate Theory Based Prediction of the Behavior of RC Slabs. Master Tthesis, vol. College of Engineering,University of Salahaddin-Erbil.2020

[15] Sharif SM, Ahmed S. Theoretical Investigation of Stresses and Displacement in RC Rectangular Slab. American Scientific Research Journal for Engineering, Technology, and Sciences (ASRJETS). 2019; 57(1): 62-76.

[16] TAHA D, ELMARDI OM. THEORIES ON LAMINATED COMPOSITE PLATES. 2017.