

Solving Linear Fractional Programming Problems Via Revised Simplex Method

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Abstract: When using fractional programming, the numerator to denominator ratio serves as the goal function. Due to the challenges' application in finance and Business Scheduling, manufacturing scheduling, hospital and health care preparation, and other areas, there has been a great deal of study and interest in these kinds of issues. Under a set of linear constraints, linear fractional programming, or LFP, aims to maximise a quotient of two linear functions. Many methods for resolving linear fractional programming issues possessed been developed in the past few years. In this research, we defined the updated simplex technique, used it to resolve linear fractional programming issues, and proposed an algorithm for it. We also employed the modified method to solve LFP problems. To demonstrate the effectiveness of the approach, many numerical cases are resolved, shown, and the outcomes are compared. Results from the updated simplex approach were shown to be more rapid and efficient than those from the modified method.

Keywords: Fractional Programming Problem; Modified Simplex Method; Revised Method.

1. Introduction

A specific class of non-linear programming issues known as linear fractional programming (LFP) problems use linear functions as constraints and the quotient of two linear functions as the objective function. Fractional models are encountered in real-world scenarios involving decision-making, including hospital and health care scheduling, economic and business organizing, and building schedule. Several techniques may be found in the literature. Pandian and Jayalakshmi presented a denominator objective limitation approach based on the simplex technique for solving LFP problems. Additionally, a decomposition-restriction approach, developed from the principle of decomposition and the method of denominator objective restriction, is suggested to find an optimum fuzzy solution for completely fuzzy LFP problems [1]. Kumari suggested a technique for resolving issues with linear fractional programming. An efficient selection of incoming vectors is the foundation of the algorithm. We begin with a basic, workable answer and work to improve it until we arrive at the best possible option [2]. Sulaiman and Asreen established a multi-objective abstract algorithm (MOFPP) for the solution method used by Chandra Sen. Additionally, they used the mean and median of the objective values as well as suggested problem-solving techniques to resolve this problem (MOFPP) [3]. Mustafa and Sulaiman presented an objective function for a LFP problem with rough interval coefficients (RICs). It demonstrates how the variable transformations may be used to change the LPP with RICs in the objective function into a LPP with RICs [4]. Mahmood and Sulaiman utilised to transform multi-objective linear programming (MOLPP) concerns into single-objective LPPs (SOLPPs). In order to achieve this, there is a method available. and recommended a solution for MOLPPs, which have been assessed using Excel Solver on numerical examples [5].

To further this study, we investigate the revised method, apply it to solving linear fractional programming problems, and provide an algorithm for it. We also Utilize the modified method to resolve linear fractional programming problems. Exam validity will be evaluated in relation to the two results.

2. Preliminaries

A few basics definition utilized in this research.

2.1 Linear Fractional Programming Problems [1, 6]

Consider the following Linear Fractional Programming problem:

$$(P) \text{ Maximise } Z = \frac{c^T x + \alpha}{d^T x + \beta},$$

$$\text{Sub. to } \mathcal{A}X \leq B, X \geq 0$$

where X, C and D are n dimensional vectors and α, β are scalars.

It is considered that the collection of feasible solutions to the problem (P), $S = \{X \in R^n: \mathcal{A}X \leq b, X \geq 0\}$ has a boundary and is not empty.

2.2 Revised Simplex Methods [7]

The revised technique is an enhancement of the simplex method. In the simplex method, all entries are computed to finalize a simplex table, however in the revised technique, just the essential details needed for calculating the existing simplex iteration is most recent and retained. The updated simplex approach is an effective technique for addressing several issues characterized by a substantial quantity of decision variables and restrictions, including inventory management, dietary optimization, product assortment, and trim loss challenges.

3. Proposed Methods

3.1 Modified Simplex Method

Dantzig created the simplex method in 1947. Its offers methodical approach that works by iteratively going from a single workable, fundamental answer (one vertex) to another in a way that maximises the objective function's value. The process of moving back and forth between vertices is repeated. There is no basis that can ever be duplicated, and there is no need to revisit a covered vertex if the goal function is improved at each hop. The procedure must result in the ideal vertex in a limited quantity of steps, since there are a limited number of vertices.

The simplex algorithm is a methodical, iterative process for resolving linear programming issues. It is composed of:

1. Attempting to solve constraint equations with a simple, workable answer.
2. Testing if is an optimum solution.
3. Enhancing the initial trial answer by a collection of guidelines, then continuing the procedure until an ideal solution is produced [8, 9].

3.2 New Revised Simplex Method

A linear fractional programming problem is formulated as follow

$$\text{Maximise } Z = \frac{c^T x + \alpha}{d^T x + \beta},$$

Sub. to

$$\begin{aligned} \mathcal{A}X &\leq B, \\ X &\geq 0 \end{aligned}$$

First write the above problem in standard form

$$(1) \quad \text{Max } Z = \frac{c_1x_1 + c_2x_2 + \dots + c_nx_n + 0x_{n+1} + 0x_{n+2} + \dots + 0x_{n+m}}{d_1x_1 + d_2x_2 + \dots + d_nx_n + 0x_{n+1} + 0x_{n+2} + \dots + 0x_{n+m}}$$

Sub. to

$$(2) \quad \begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n + x_{n+1} &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n + x_{n+2} &= b_2 \\ \vdots & \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n + \dots + x_{n+m} &= b_m \end{aligned}$$

And

$$(3) \quad x_1, x_2, \dots, x_{n+m} \geq 0,$$

where the beginning basis matrix B^{-1} is an $m \times m$ identity matrix.

The goal function (1) is also treated as if it were an additional constraint in the revised form, where z is maximally big and has unconstrained sign.

Therefore, (1) and (2) may be expressed compactly as follows:

$$(4) \quad \begin{aligned} Z_1 - c_1x_1 - c_2x_2 - \dots - c_nx_n - 0x_{n+1} - 0x_{n+2} \dots - 0x_{n+m} &= \alpha \\ Z_2 - d_1x_1 - d_2x_2 - \dots - d_nx_n - 0x_{n+1} - 0x_{n+2} \dots - 0x_{n+m} &= \beta \\ a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n + x_{n+1} &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n + x_{n+2} &= b_2 \\ \vdots & \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n + \dots + x_{n+m} &= b_m \end{aligned}$$

and $x_1, x_2, \dots, x_{n+m} \geq 0$, where α and β are constants.

In the revised table, write the Δ_j as follow

$$\Delta_j = (\text{first row of } B_1^{-1}) \times a_j$$

If all $\Delta_j \geq 0$, we get the optimal solution otherwise find the most negative value to determine the incoming vector.

To remove a variable from the basis (key row), select it. Determine the following: $y_k^{(1)} = B_1^{-1}a_k^{(1)} = a_k^{(1)}$; ($k = 1$). The best solution is unbounded if all $y_{ik} \leq 0$. However, the variable to be eliminated from the basis is identified by computing the ratio if at least one $y_{ik} > 0$:

$$\frac{x_{Br}}{y_{rk}} = \text{Min}_i \left\{ \frac{x_{Bi}}{y_{ik}}; y_{ik} > 0 \right\}$$

In other words, the vector $\beta_r^{(1)}$ is chosen to exit the basis and proceed to the fourth step.

Revise the existing resolution. In order to update the original table, remove the basic variable $x_r (= \beta_r^{(1)})$ from the basis and add a non-basic variable $x_k (= a_k^{(1)})$ to the basis.

Repeat steps 3 and 4 until an ideal solution is reached or there is an indication for an unbounded solution.

4. Algorithms

4.1 Algorithm for modified simplex method

An algorithm for solving LFPP via modified simplex approach may be stated as follows:
Step 1: build the conventional form of the problem, by inserting slack and fake variables to limitations, and build beginning simplex table.

Step 2: In the initial simplex table, write the Δ_j row as follows:

$$\Delta_j = Z_2 * \Delta_{j1} - Z_1 * \Delta_{j2}$$

Step 3: To determine the answer, use the simplex method.

Step 4: Assess the solution's viability in step 3, if it is, go to step 5, if not, eliminate the impossibility using the dual simplex approach.

Step 5: Verify the optimality of the solution. The solution is ideal if all $\Delta_j \geq 0$, if not, go on to step3 [10].

4.2 Algorithm for new revised simplex method

The steps below may be used to sum up the revised technique:

Step 1: Write the given issue in standard form. Consider the goal function as one of the constraints when expressing the provided issue in the revised model. If necessary, add the slack and excess variables to the inequalities to make them equal.

Step 2: In the revised table, write the Δ_j as follow:

$$\Delta_j = (\text{first row of } B_1^{-1}) \times a_j$$

If all $\Delta_j \geq 0$, we get the optimal solution otherwise find the most negative value to determine the incoming vector.

Step 3: To remove a variable from the basis (key row), select it. Determine the following: $y_k^{(1)} = B_1^{-1} a_k^{(1)} = a_k^{(1)}$; ($k = 1$).

The best solution is unbounded if all $y_{ik} \leq 0$. However, the variable to be eliminated from the basis is identified by computing the ratio if at least one $y_{ik} > 0$:

$$\frac{x_{Br}}{y_{rk}} = \text{Min}_i \left\{ \frac{x_{Bi}}{y_{ik}}; y_{ik} > 0 \right\}$$

In other words, the vector $\beta_r^{(1)}$ is chosen to exit the basis and proceed to the fourth step.

Step 4: Revise the existing resolution. In order to update the original table, remove the basic variable $x_r (= \beta_r^{(1)})$ from the basis and add a non-basic variable $x_k (= a_k^{(1)})$ to the basis.

Repeat steps 3 and 4 until an ideal solution is reached or there is an indication for an unbounded solution.

5. Numerical Examples

Problem 1: By Modified Simplex Method

Solving the problem using a modified method is examined, and the methodology of the solution is provided as follows:

$$\text{Max } Z = \frac{5x_1 + 6x_2}{2x_2 + 7}$$

Sub. to

$$2x_1 + 3x_2 \leq 6$$

$$2x_1 + x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

Solution:

$$Z_1 = 5x_1 + 6x_2$$

$$Z_2 = 2x_2 + 7$$

Sub. to

$$\begin{aligned} 2x_1 + 3x_2 + S_1 &= 4 \\ 2x_1 + x_2 + S_2 &= 1 \\ x_1, x_2, S_1, S_2 &\geq 0 \end{aligned}$$

B.S.	C_{B1}	C_{B2}	X_B	x_1	x_2	S_1	S_2	Min ratio
S_1	0	0	6	2	3	1	0	6/3=2
S_2	0	0	3	2	1	0	1	3/1=3
	$Z_1=0$		Δ_{j1}	-5	-6	0	0	
	$Z_2=7$		Δ_{j2}	0	-2	0	0	
			Δ_j	-35	-42	0	0	

$$\Delta_j = Z_2 \cdot \Delta_{j1} - Z_1 \cdot \Delta_{j2}$$

B.S.	C_{B1}	C_{B2}	X_B	x_1	x_2	S_1	S_2
x_2	6	2	$\frac{3}{2}$	0	1	$\frac{1}{2}$	$\frac{-1}{2}$
x_1	5	0	$\frac{3}{4}$	1	0	$\frac{-1}{4}$	$\frac{3}{4}$
	$Z_1 = \frac{51}{4}$		Δ_{j1}	0	0	$\frac{7}{4}$	$\frac{3}{4}$
	$Z_2 = 10$		Δ_{j2}	0	0	1	-1
			Δ_j	0	0	$\frac{19}{4}$	$\frac{81}{4}$

Since all $\Delta_j \geq 0$, the optimal is $x_1 = \frac{3}{4}$, $x_2 = \frac{3}{2}$, $Z_1 = \frac{51}{4}$, $Z_2 = 10$ and $Max Z = \frac{51}{40}$

By Revised Simplex Method:

Solving the problem using revised method is examined, and the methodology of the solution is provided as follows:

$$Max Z = \frac{5x_1 + 6x_2}{2x_2 + 7}$$

Sub. to

$$\begin{aligned} 2x_1 + 3x_2 &\leq 6 \\ 2x_1 + x_2 &\leq 3 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Solution:

$$\begin{aligned} Z_1 - 5x_1 - 6x_2 &= 0 \\ Z_2 - 2x_2 &= 7 \\ 2x_1 + 3x_2 + S_1 &= 6 \\ 2x_1 + x_2 + S_2 &= 3 \\ x_1, x_2, S_1, S_2 &\geq 0 \end{aligned}$$

First Table

Basic Solution	e_1	e_2	a_4	a_5	x_B	x_k	Min $\frac{x_B}{x_k}$
	Z_1	Z_2	S_1	S_2			
Z_1	1	0	0	0	0	-6	×
Z_2	0	1	0	0	7	-2	×
S_1	0	0	1	0	6	3	2 ←
S_2	0	0	0	1	3	1	3

Non-Basic Variables

a_1	a_2
-5	-6
0	-2
2	3
2	1
-5	-6

↑

Final Table

Basic Solution	e_1	e_2	a_4	a_5	x_B
	Z_1	Z_2	x_1	x_2	
Z_1	1	0	$\frac{7}{4}$	$\frac{3}{4}$	$\frac{51}{4}$
Z_2	0	1	1	-1	10
x_2	0	0	$\frac{1}{2}$	$\frac{-1}{2}$	$\frac{3}{2}$
x_1	0	0	$\frac{-1}{4}$	$\frac{3}{4}$	$\frac{3}{4}$

a_1	a_1
0	0
0	0
0	1
1	0
$\frac{3}{4}$	$\frac{7}{4}$

Since all $\Delta_j \geq 0$, the optimal is $x_1 = \frac{3}{4}$, $x_2 = \frac{3}{2}$, $Z_1 = \frac{51}{4}$, $Z_2 = 10$ and $Max Z = \frac{51}{40}$

Problem 2: By Modified Simplex Method

Solving the problem using a modified method is examined, and the methodology of the solution is provided as follows:

$$Max Z = \frac{3x_1 + x_2 + x_3 + 1}{x_1 + 2x_2 + 3x_3 + 2}$$

Sub. to

$$\begin{aligned} 2x_1 + x_2 + 3x_3 &\leq 4 \\ x_1 + 2x_2 + x_3 &\leq 1 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Solution:

$$\begin{aligned} Z_1 &= 3x_1 + x_2 + x_3 + 1 \\ Z_2 &= x_1 + 2x_2 + 3x_3 + 2 \end{aligned}$$

Sub. to

$$\begin{aligned} 2x_1 + x_2 + 3x_3 + S_1 &= 4 \\ x_1 + 2x_2 + x_3 + S_2 &= 1 \\ x_1, x_2, x_3, S_1, S_2 &\geq 0 \end{aligned}$$

B.s.	C_{B1}	C_{B2}	x_B	x_1	x_2	x_3	S_1	S_2	Min ratio
S_1	0	0	4	2	1	3	1	0	4/2=2
S_2	0	0	1	1	2	1	0	1	1/1=1 ←
	$Z_1=1$		Δ_{j1}	-3	-1	-1	0	0	
	$Z_2=2$		Δ_{j2}	-1	-2	-3	0	0	
			Δ_j	-5	0	1	0	0	

↑

$$\Delta_j = Z_2 \cdot \Delta_{j1} - Z_1 \cdot \Delta_{j2}$$

B.s.	C_{B1}	C_{B2}	X_B	x_1	x_2	x_3	S_1	S_2
S_1	0	0	2	0	-3	1	1	-2
x_1	3	1	1	1	2	1	0	1
	$Z_1=4$		Δ_{j1}	0	5	2	0	3
	$Z_2=3$		Δ_{j2}	0	0	-2	0	1
			Δ_j	0	15	14	0	5

Since all $\Delta_j \geq 0$, the optimal is $x_1 = 1, x_2 = x_3 = 0, Z_1 = 4, Z_2 = 3$ and $Max Z = \frac{4}{3}$

By Revised Simplex Method

Solving the problem using revised method is examined, and the methodology of the solution is provided as follows:

$$Max Z = \frac{3x_1 + x_2 + x_3 + 1}{x_1 + 2x_2 + 3x_3 + 2}$$

Sub. to

$$2x_1 + x_2 + 3x_3 \leq 4$$

$$x_1 + 2x_2 + x_3 \leq 1$$

$$x_1, x_2, x_3 \geq 0$$

Solution:

$$Z_1 - 3x_1 - x_2 - x_3 = 1$$

$$Z_2 - x_1 - 2x_2 - 3x_3 = 2$$

$$2x_1 + x_2 + 3x_3 + S_1 = 4$$

$$x_1 + 2x_2 + x_3 + S_2 = 1$$

$$x_1, x_2, x_3, S_1, S_2 \geq 0$$

First Table

Basic Solution	e_1	e_2	a_4	a_5	X_B	X_k	Min $\frac{x_B}{x_k}$
	Z_1	Z_2	S_1	S_2			
Z_1	1	0	0	0	1	-3	×
Z_2	0	1	0	0	2	-1	×
S_1	0	0	1	0	4	2	2
S_2	0	0	0	1	1	1	1

Non-Basic Variables

a_1	a_2	a_3
-3	-1	-1
-1	-2	-3
2	1	3
1	2	1

-3 -1 -1
 ↑

Final Table

Basic Solution	e_1	e_2	a_4	a_5	x_B
	Z_1	Z_2	S_1	x_1	
Z_1	1	0	0	3	4
Z_2	0	1	0	1	3
S_1	0	0	1	-2	2
x_1	0	0	0	1	1

a_1	a_2	a_3
0	-1	-1
0	-2	-3
0	1	3
1	2	1
3	5	2

Since all $\Delta_j \geq 0$, the optimal is $x_1 = 1, x_2 = x_3 = 0, Z_1 = 4, Z_2 = 3$ and $Max Z = \frac{4}{3}$

6. Comparison of the numerical results is shown in the following

6.1 Comparison between two methods (Modified Method, Revised Method)

Methods	Example 1		Example 2	
	$Max Z$	(x_1, x_2)	$Max Z$	(x_1, x_2, x_3)
Modified Method	$\frac{4}{3}$	(1,0)	$\frac{51}{40}$	$(\frac{3}{4}, \frac{3}{2})$
Revised Method	$\frac{4}{3}$	(1,0)	$\frac{51}{40}$	$(\frac{3}{4}, \frac{3}{2})$

In the table above, we compare the outcomes. It is observed that the objective function values in Examples (1) and (2) provide the same results when solved using the modified method and the revised method.

6.2 Comparison Analysis: Revised Method vs. Modified Method

This part explores a comparison, between the Revised Technique and the Modified Technique for solving LFPPs with a focus, on the following factors;

Key Contrasts, in Algorithms; We emphasize the differences in the processes focusing on how the simplex tableau is represented and updated bases along with the criteria, for optimality.

Analyzing Computational Efficiency; We compare the time it takes for both approaches to solve LFPPs of sizes and complexities supported by experiments we conducted.

Storing Data, in Memory Usage Assessment; We examine the memory required to hold information while running each procedure with a focus, on how problem scale influences it.

Let’s talk about the pros and cons of each approach when it comes to tackling LFPPs – thinking about things, like how they work on computers and how easy they’re to put into practice for various types of issues.

7. Conclusion

We have described in this study a method for solving LFP problems using the revised technique. We can efficiently arrive at the optimum answer by transforming the function into a linear representation and using the typical simplex approach. This method may be used to several situations involving ratio optimisation. Additionally, specialised software tools simplify this procedure for experts overcoming programming obstacles.

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